OPTIMAL LIMIT STATE DESIGN OF REINFORCED CONCRETE STRUCTURES

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
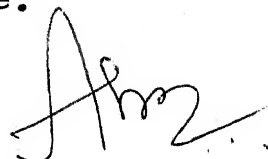## DOCTOR OF PHILOSOPHY

by

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to the

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CERTIFICATE

This is to certify that the thesis "Optimal Limit
State Design of Reinforced Concrete Structures" submitted
by Shri A.V. Subramanyam in partial fulfilment of the
requirements for the degree of Doctor of Philosophy of the
Indian Institute of Technology, Kanpur, is a record of
bonafide research work carried out by him under my super-
vision and guidance. The work embodied in this thesis has
not been submitted elsewhere for a degree.

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A_{dome} surface area of the domed roof

A_h area of hoop reinforcement for the ring beam

A_{sf} area of side face reinforcement for the T-beam

A_{st} area of tension reinforcement for the T-beam

A_{sv} average area of shear reinforcement for the T-beam

a a constant

a_{cr} distance from point under consideration to the surface of the nearest longitudinal bar

a_h area of hoop reinforcement per unit width

a_{min} minimum area of reinforcement per unit width

a_p area of reinforcement in the floor slab per unit width

a_{s1} area of reinforcement per unit width in the end span of the slab

a_{s2} area of reinforcement per unit width over the interior supports

a_{s3} area of reinforcement per unit width in the interior span

a_v area of vertical reinforcement per unit width

b a constant; width of the ring beam

b_t width of the member at the level of tension reinforcement

b_w width of rib

c_1, c_2 etc.   constants of integration

c_{min} minimum cover to reinforcement

D_p flexural rigidity of the plate

D_w flexural rigidity of the wall

d effective depth of the member

d_b effective depth of the beam

d_n depth of neutral axis

d_s effective depth of the slab

E modulus of elasticity of concrete

$\bar{E}$ error vector

F objective function

F', F_b etc.   objective function of parts of the structure

f dome parameter; function of a single variable

f_{bs} maximum bond stress in the beam

f_{dbs} design bond stress

f_{max} maximum allowable tensile stress in the reinforcement

f_s stress in the reinforcement at service loads

f_y characteristic strength of reinforcement

g constraint

H total height of the tank wall

$[H]$ a positive definite symmetric matrix

H_1, H_2, H_3 reference heights of the tank wall

h depth of the member; error in the value of the root

h_b depth of the beam

h_f	thickness of the floor slab of the elevated tank
h_s	thickness of the slab
I	second moment of area
[J]	Jacobian matrix
j	constraint number
k	cycle number
k_{hh}^b etc.	stiffness factors of the beam
k_{hm}^d etc.	stiffness factors of the dome
k_{mm}^p	stiffness factor of the plate
k_{hh}^w etc.	stiffness factors of the wall
l	length of the shell
l_b	span of the beam
l_s	span of the slab
l_1, l_2	reference lengths of the tank wall
M	bending moment due to ultimate loads
[M]	an updating matrix
M_u	moment of resistance of the beam
m	number of design variables
m_u	moment of resistance of slab or shell per unit width
m_u'	moment of resistance of the shell per unit width to resist negative moment
[N]	an updating matrix
N_h	hoop membrane force in the dome
N_m	meridional membrane force in the dome
N_x	longitudinal membrane force in the shell

N_θ	hoop membrane force in the shell
n	number of constraints
n_c	hoop compression capacity per unit width
n_t	hoop tension capacity per unit width
P	line load on the plate
p_c	**collapse pressure**
p_x	water pressure at section x
$\bar{Q}$	gradient difference vector
Q_u	ultimate line load
Q_x	shear force in the shell at section x
q	distributed load
q_d	design load per unit plan area over the dome
q_k	characteristic imposed load per unit area
q_u	distributed load required to cause collapse
R	radius of the tank wall
R_d	radius of the dome
R_t	radius of the tower
R_1	ratio of costs of one m^3 of finished concrete to one newton of reinforcement
R_2	ratio of costs of one m^2 of formwork to one newton of reinforcement
r	penalty parameter; radial distance
$\bar{s}$	search direction
t	thickness of the tank wall
t_d	thickness of the domed roof
t_{min}	minimum thickness

V	shear force due to ultimate loads
V_u	shear capacity of the beam
w	radial displacement of the tank wall
w_{cr}	estimated width of surface flexural cracks
w_{crp}	permitted width of crack
$\bar{X}$	vector of design variables
x	reference length; variable
x_d	translation of the dome edge
x_w	translation of the bottom edge of tank wall
y	distance from the neutral axis to the level of point under consideration
y_{max}	maximum deflection in the beam
α	step length; ratio of supporting tower and tank wall radii
α^*	minimizing step length
β	degree of fixity
γ_s	specific weight of steel
γ_w	specific weight of water
Δ	virtual displacement
ε_m	mean strain
ε_s	strain in steel at service loads
Θ	slope of the elastic curve
Θ_p	rotation of the plate
Θ_w	rotation of the bottom edge of the tank wall due to joint forces
Θ_1	rotation of joint 1

θ_2 rotation of joint 2

κ_x generalized strain associated with M_x

λ dome parameter

λ_θ generalized strain associated with N_θ

μ tank wall parameter

ξ modification factor

ϕ modified objective function; angle

ϕ_o semi-central angle

ψ_d free rotation of the dome edge

ψ_w free rotation of the tank wall

SYNOPSIS

OPTIMAL LIMIT STATE DESIGN OF
REINFORCED CONCRETE STRUCTURES

(A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy by A.V. Subramanyam to the Department of Civil Engineering, Indian Institute of Technology, Kanpur, India)

Reinforced concrete structures, in general, are designed either by the permissible stress method based on elastic theory or by the ultimate load theory with a suitable load factor. It is recognized that **elastic theory** is better suited to predict the **response** of a structure to service loads and the **ultimate load theory** to determine the **collapse** modes and the associated loads. Randomness involved in basic parameters has rendered the **deterministic** approach of these methods to safety, obsolete. These considerations set the stage for development of a more **rational** and **sound** design philosophy, as a consequence of which the **limit state theory** has emerged. On the recommendations of the International Standard, codes of practice for design of reinforced concrete structures are being modified to conform to the limit state philosophy.

In the present study, optimal design of some reinforced concrete structures using the limit state method is investigated. The structures chosen are, **T-beam floors**

and **cylindrical water** tanks , surface as well as overhead. These structures are built, around the globe, in large numbers and hence optimization will be quite meaningful. Besides, the structures considered herein, also **highlight** different limit states that basically govern their design.

The resistance of the foregoing structures to ultimate loads is obtained from limit analysis and response to service loads predicted by elastic analysis. Application of these analyses for the design of slabs and T-beams associated with a T-beam floor is no doubt well understood, as a result of which routine procedures are readily available for the design of these structural elements. However, **optimal** design of T-beam floors, considering **all** the relevant limit states and unit costs of different materials and formwork, has been investigated for the **first** time.

Elastic analysis of surface water tanks, cylindrical in shape and with or without a roof, is normally carried out by assuming the tank walls to be either fixed or hinged to the floor slab. It has been recognized that the **actual** restraint at the base corresponds to neither of these conditions, but depends on the amount of rotation of the base, which in turn is a function of the behaviour of the subgrade. In order to account for this variability in restraint, a method of analysis has been developed in which **any** degree of fixity can be assigned.

For a proper application of the limit state theory, resistance of water tanks to ultimate loads is to be ascertained. Therefore, various collapse mechanisms of reinforced concrete cylindrical shells subjected to a hydrostatic pressure, adopting a suitable failure criterion, are investigated. This leads to a value of limit load which is **exact** in the sense that it satisfies both equilibrium and failure criterion.

All the foregoing analyses are incorporated in a **nonlinear programming** method of seeking the optimum solution. Designs have been carried out to conform to CP 110 in the case of T-beam floors and BS 5337 for water tanks. Extensive **parametric studies** have been conducted over a wide range of material and formwork costs. For various values of span of slab, span ratio of beam to slab and imposed load, optimal designs of T-beam floors are arrived at, by considering different **grades** of concrete and adopting high yield strength deformed bars as reinforcement. The capacity of tank and **degree of fixity** at the base are treated as parameters in the study of optimal design of water tanks.

The current study has demonstrated that **substantial** reduction in thicknesses of members and areas of reinforcement is possible which results in considerable savings in the cost of the structures. Some of the other salient observations are those regarding the importance of deflection considerations in the optimal design of slabs, optimal

values of upper bound on the depth of T-beam and degree of

fixity for the surface water tanks.

In order to bridge the gap between research

activity and professional practice, design charts have been

developed, from which optimal values of design variables can

be picked from the wide range of data considered for all

these structures. Thus, the designer will be able to arrive

at an **optimal** design which conforms to the relevant code of

practice and without recourse to a powerful computer. The

effectiveness of optimization and the method of utilizing

the design charts are illustrated with a few examples.

CHAPTER 1

INTRODUCTION

1.1 General

The aim of a structural engineer is to design safe, economic, and aesthetically pleasing structures, which remain serviceable throughout their design life. Permissible stress method and ultimate load theory, either of which is generally adopted for design of reinforced concrete structures at present, have been found to be inadequate to achieve such an objective. It has also been recognized that, in order to take into account the variability in the basic design parameters, a probabilistic approach to safety has to be adopted. These aspects have been examined by various national and international committees and a semi-probabilistic limit state method recommended.

A satisfactory design can no doubt be obtained through applications of the limit state philosophy. But, in order to ensure economy and efficiency in such designs, optimization techniques have to be incorporated in the design process. This is all the more important in view of the depleting resources.

The foregoing considerations have motivated this study dealing with optimal design of some reinforced concrete structures by the limit state method.

1.2 Structures Chosen for the Study

The analysis and methodology developed in this study can be applied to obtain optimal designs of beams, slabs and storage structures like water tanks and silos. These represent one-dimensional and two-dimensional structures having plane as well as curved surfaces. However, detailed investigations have been limited to the study of T-beam floors and cylindrical water tanks. One of the reasons for choosing them is that they are among the most commonly built structures in the field of reinforced concrete construction. These structures fall into what may be termed standard types which are repeated over a large number of times, with the result that any saving that can be made in the design of these structures gets added up. Hence optimization will be meaningful.

Another reason for their choice is that the optimum design of these structures is governed by different limit states. The limit state of flexure is the most critical limit state in the optimum design of T-beams, while limit state of deflection controls optimum design of slabs. The governing limit state in the optimum design of water tanks is the limit state of cracking. Therefore, a study of the optimum design of these structures helps to understand the applicability of different limit states in a better perspective.

1.3   Selective Literature Review

The current study encompasses many fields such as the limit state philosophy, limit analysis of structures, response of reinforced concrete structures to service loads, and optimization. Literature review on some of these fields has been well documented and available even in some textbooks (Jones and Wood, 1967; Rao, 1978). Hence only a selective review of literature, on the development of limit state philosophy and application of optimization techniques for the design of structures under consideration, will be made.

1.3.1   Development of limit state philosophy

1.3.1.1   Historical background

Ever since reinforced concrete was first put into use in 1850, there has been a constant endeavour to understand the behaviour of reinforced concrete structures better and to improve upon its designing methods. No doubt, initially the emphasis was to successfully use this material for different types of structures. But once its use was accepted, thought was given to methods of improving the strength of concrete and steel as well as for better design procedures.

Two mathematical models have been extensively used for the design of reinforced concrete structures and both are deterministic in concept. In the elastic or working stress approach, the structure is assumed to act as a linear

elastic body under the action of specified loads. The members of the structure are so proportioned that the maximum stresses in concrete and steel, produced in the most critical section of the structure, for the worst combination of loads, do not exceed the permissible values. These permissible values are obtained as a fraction of the specified strengths of concrete and steel. Many structures have been successfully designed by this method and it is still popular with some designers. The objections to this method are not that it is unsafe, but that it is physically unreal and leads to unnecessarily conservative designs with varying degrees of safety against collapse. The maximum stress may occur at only one point in the whole structure which means that safety is being assessed by looking at a very local effect. The stresses predicted by elastic analysis are seen to be in poor agreement with actual stresses even for steel structures. Obviously for concrete structures, the elastic analysis would be much less valid because concrete is never truly elastic.

Dissatisfaction with this elastic model led, in the post-war years, to the development of the earlier method of ultimate loads which had been abandoned with the advent of the theory of elasticity. According to the ultimate load theory, also termed as plastic analysis and limit analysis, failure does not occur because of the stress reaching the maximum value at one point. On the other hand, it takes into account the inelastic behaviour of the materials and

redistribution of stresses. Tests have confirmed that the mode of collapse and loads producing it are remarkably well predicted by this method. The margin of safety against collapse, called the load factor, can be chosen as per the requirement. Another important contribution of this method is that it has diverted the attention from the microscopic stress concept to the macroscopic overall behaviour of the structure. Although this is an excellent model for finding the strength of a structure, it has a serious limitation that it cannot predict the response of the system to service loads. In order to get an idea of deflection or crack width at service loads, elastic model is still being used.

As pointed out earlier, both these models are deterministic in concept. It had long been recognized that the loads acting on structures, strengths of construction materials, quality control during construction are all variable and a rational value for the safety of the structure can be obtained only through a probabilistic approach. These considerations led to the development of the limit state theory.

1.3.1.2   The concept of safety

Society has for long demanded a degree of safety in structures which is out of proportion to the risks it accepts in other areas of life. Therefore the design procedures have generally resulted in conservative, over-designed structures. As this concept of safety is deep rooted in

engineering design, the notion of probability of failure, even if it is of the order of 10^{-7}, is repulsive to the majority of present day engineers. This is reflected in Baker's statement (1966) that except for absolutely unforeseeable events, a zero probability of failure has to be demanded from the designer. But, to quote Prof. Freudenthal, 'the difference between safe and unsafe design is in the degree of risk considered acceptable, not in the delusion that such a risk can be completely eliminated'.

A probabilistic approach to the structural safety, taking into account expected variations in load and strength, was first introduced into structural design in the aircraft industry in the late 1940s. Following the attempts by Freudenthal (1947, 1948) to derive the safety factor from statistical variations of the design characteristics, concept of safety was extensively considered, particularly by Pugsley (1951) in England, Prot and Levi (1951) in France, Balaca and Torroja (1950) in Spain and Johnson (1953) in Sweden. A composite international body of research workers and practising designers known as the European Committee of Concrete, CEB, was set up in 1953 with the object of providing a sound philosophy of design, together with recommendations on the detailed aspects of design based upon experimental and theoretical research. The CEB published over forty bulletins synthesizing research work and also the 'Recommendations for an International Code of Practice for Reinforced Concrete' (1964).

The question of what constitutes an acceptable risk or probability of failure is very controversial and merits particular attention (Rowe et al., 1965). It could be based either on a direct probability approach which specifies a numerical value for probability of failure or an economic approach which considers the trade off between margin of safety and cost of risk (Freudenthal, 1956). It was suggested by the CEB that the risks of structural failure should be considered on a cost basis by assessing the insurance premium necessary to cover the consequences of the various forms of structural failure envisaged. According to the International Standard (TC98, 1973), it is not generally required that structures should be designed to withstand effects of exceptional events like wars. In case of certain other events, like accidents or earthquakes, when their frequency is ill defined, the designer should ensure that the risks associated with such events are limited. In assessing the optimum cost of construction, the factors to be considered are the initial cost of construction, the maintenance cost, the cost of losses, both human and material, arising from failure associated with the degree of risk accepted, general social inconveniences resulting from failure, moral considerations with respect to human life, and adverse reaction from the society due to the failure.

In the classical reliability theory, if the random resistance, R, and the random load, S, are described by

their known probability density functions, and the uncertainities are strictly those associated with the inherent randomness, then the measure of risk is the probability of failure event R ≤ S. There are a number of reasons why classical reliability theory cannot be applied to the design of reinforced concrete structures (Kemp, 1973). Some of the important ones are the following:

(1)   Prototype testing of complete structures is only very rarely possible and the designer is compelled to rely on the study of mathematical and physical models which inevitably involve significant idealization.

(2)   For the acceptable levels of risks, which would be normally less than 10^{-5}, the failure probability could be quite sensitive to the behaviour of the extreme tails of the distribution functions. Data available are limited to central range of the variates.

(3)   The problem of calculating the probability of failure of any real structure is quite complicated because of the multiple loads and multiple modes of failure, usually with complex correlation between them. The assessment of joint probability of failure then becomes a difficult analytical task which has so far been solved for a few simple problems only.

(4)   There are some uncertainties associated with design and construction processes which are not random in nature and hence can only be taken into account by a

suitable factor of safety based on sound judgment.

(5)   Current legal, professional and social responsibilities

do not permit use of an explicit probability of failure.

The alternative approach to which the profession
is now moving is a compromise between the classical reliab-
ility theory and deterministic methods. In the limit state
philosophy, failure is interpreted with respect to some pre-
determined limit state. Failure probabilities need not,
however, appear explicitly in the routine design equations
(Ang and Cornell, 1974).

1.3.1.3   Definition of limit states

A structure, or a part of a structure, is rendered
unfit for use when it reaches a particular state, called a
'limit state', in which it ceases to fulfil the function, or
to satisfy the condition, for which it was designed (TC98,
1973).

It is convenient to group limit states into three
major categories as follows:

(1)   ultimate limit states: those corresponding to maximum

load carrying capacity and may be caused by rupture of

critical sections of the structure, excessive plastic

deformation leading to collapse, instability etc.;

(2)   serviceability limit states: those corresponding to

excessive deformation, wide cracking, excessive

vibrations, undesirable damage etc.;

(3)   Other limit states: those corresponding to fatigue,
       fire, lightning, explosions etc.

1.3.1.4   Limit state philosophy

Thought was given to the foregoing aspects in
Soviet Union as early as 1930 and the limit state approach
was embodied in the Russian Code in 1954 (Zalesov, 1973).
This philosophy was adopted and elaborated by the CEB and
formed the basis for that committee's recommendations (1964).
The International Standard, aiming at unification of different
methods of structural calculations and ensuring the safety
of structures, has recommended a semi-probabilistic limit
state method.

Thus, probabilistic concepts have been for the
first time accepted explicitly in the design. It is assumed
that the statistical distribution of loads and material
strengths are available. 'Characteristic values' of loads
and material strengths are then selected with specified
probabilities against higher load or lower strength respec-
tively. The practice of using a global factor of safety has
been replaced by a probability-based partial safety factor
format. The design loads are then computed by multiplying
the characteristic loads by specified safety factors which
vary with the degree of seriousness of the particular limit
state being reached, probability of two or more loads
occurring together, and the reliability of the structural

theories being used. Similarly, the design stresses are obtained by dividing the characteristic strengths by another partial safety factor which accounts for the difference between the strengths of controlled specimens and the material in the real structure, and any other possible but unpredictable reduction in strengths. With these modified loads and material strengths, safety of the structure is investigated for the relevant limit states.

Since code formulation is an evolutionary process, the new provisions seldom reflect abrupt changes from established ones (Ellingwood and Ang, 1974). Therefore, it is likely that initially there may not be considerable difference between the designs done in accordance with the reliability-based code and the deterministic code. But, the introduction of this method is an important step in design thinking and should stimulate the research and study necessary to obtain the additional statistical data required. With the modern communication and monitoring facilities, it should be possible to build the required data base in a comparatively short time. This will enable in choosing more appropriate values for characteristic strengths and characteristic loads.

Another important aspect of the limit state philosophy is its flexible framework; it will allow future code provisions to evolve in a more consistent and rational manner. If more information becomes available, or engineering practices change, the effect of the design safety could

be reflected by reevaluating the uncertainties. The level of safety provided by future codes may be modified and carefully controlled by adjusting the risk levels relative to those considered acceptable at that time.

1.3.2   Application of optimization techniques

1.3.2.1   Reinforced concrete slabs

Most of the studies on optimal design of slabs deal with minimization of reinforcement to resist a given moment or a given distribution of moment. The thickness of the slab is either assumed to be known or preassigned. Deflection considerations have rarely been included even among those studies which treat thickness of slab as one of the design variables.

Expressing the cost of concrete in terms of its strength and assuming a balanced design, dimensions of reinforced concrete members were obtained for least cost (Norman, 1964). Economical design of reinforced concrete slabs using ultimate strength theory was considered by Traum (1963), Friel (1974) and Brown (1975). The objective function was the cost of concrete and main reinforcement. This was modified to take care of the ultimate bending moment constraint using a Lagrange multiplier and the cost minimized. Traum considered the cost of secondary reinforcement also. One of the important conclusions was that the objective function was somewhat flat near the optimum and hence slight

departures from the optimum percentage of reinforcement did not significantly affect the cost. Srinivasan (1970) also came to a similar conclusion. Srinivasa Rao and Krishnamoorthy (1970) carried out cost studies of reinforced concrete flexural members as per IS:456 (1964). Mild steel reinforcement and different grades of concrete were used and designs carried out using both working stress method and ultimate load theory. They concluded that balanced sections were the most economical as per working stress method; for ultimate load designs, balanced sections for slabs and under-reinforced sections for beams led to the most economical design. The relative economy of one method with respect to another was found to depend on the ratio of dead load to live load. Designs were found to be insensitive to concrete strengths and relatively insensitive to cost ratio of steel to concrete.

The criterion of a stationary value in potential energy was used as an objective in the structural design (Brotchie, 1969). The dimensions of the concrete were assumed to be fixed, and only the spatial distribution of reinforcement defined by a vector considered as a variable in the design. Neglecting the effect of transverse reinforcement, a computable general result for optimal construction of reinforced concrete beams and slabs was obtained, through assumptions chiefly limited to first order theory (Kaliszky, 1968).

The problem of minimizing reinforcement in a slab of given thickness was extensively considered by

Rozvany (1966, 1968), Rozvany and Adidam (1971, 1972), Charrett and Rozvany (1972), Melchers (1971, 1975), and Mroz (1967). The method adopted was essentially minimization of the moment volume from a statically admissible moment field. Both circular plates and rectangular plates with various boundary conditions were considered. Thakkar and Sridhar Rao (1970) obtained solutions for clamped square slabs.

Minimizing the weight of main reinforcement in reinforced concrete slabs was presented in the form of mathematical programming (Datta and Ratnalikar, 1973). Areas of reinforcing bars were assumed to be proportional to bending moments in the limit state. Solutions were given for slabs with simply supported and clamped edges.

The selective active constraints technique was used for the optimal design of reinforced concrete slabs and columns, within the framework of design codes (Thakkar, 1974). Combination of high strength concrete and low strength steel for reinforced concrete slabs and minimum percentage of longitudinal reinforcement for axially loaded rectangular columns was observed to be optimum.

Gunaratnam and Sivakumaran (1978) have given curves for selecting optimum design parameters of reinforced concrete slabs subjected to uniform, triangular or parabolic distribution of moments. Limit state design conforming to CP 110 (1972) forms the basis for design. Cost of secondary reinforcement has not been included in the objective function.

The optimum solution for a section is obtained, using the
Lagrange multiplier technique, initially considering only the
ultimate limit state requirement. This is then modified, if
required, to ensure that the deflection limit state is not
reached.

1.3.2.2   Reinforced concrete beams

Optimal design studies on beams invariably consider
the limit state of collapse, only, that too limited to flexure,
in almost all the cases. Cost of formwork is ignored in most
of the studies.

Sawyer (1952) considered the effect of variation in
the depth of a singly reinforced rectangular beam on its cost.
Whitney's theory was used to find the ultimate strength. He
concluded that optimum dimensions almost never correspond to
balanced design, but depend on relative values of material
costs, material strengths, lengths of beam reinforced for
diagonal tension, and ratio of dead to live loads.

Optimum designs of reinforced concrete beams were
obtained using dynamic programming (Hill, 1966), and geometric
programming (Templeman and Winterbottom, 1973). Goble and
Moses (1975) have considered application of automated
structural optimization, using a penalty function method, to
the design of reinforced concrete beams.

An equation to find out the economical depth of
T-beam to resist a given moment, taking into account the

cost ratio of steel to concrete, was given by Jaikrishna and Jain (1971). Working stress method was adopted. Takashi Chou (1977) has obtained optimum design of T-beam section, in accordance with ACI specifications (1971), using the ultimate load theory.

Adidam et al. (1978) have considered the optimum design of T-beam and grid floors through mathematical programming. The design of T-beam has been carried out by both the working stress method and the ultimate strength method while orthotropic plate theory has been adopted for the design of grid floors.

1.3.2.3 Reinforced concrete water tanks

Wilby (1977a) in his extensive literature survey on structural analysis of reinforced concrete tanks has concluded that very little work has been done on the optimization of tanks using either elastic or plastic models.

Melchers and Rozvany (1970) solved the problem of minimizing the reinforcement in cylindrical reinforced concrete water tanks of a given thickness using Prager-Shield optimality condition. The important aspect of cracking was ignored by them and steel stressed to its yield point in the plastic analysis.

Nielsen (1974) gave a procedure of finding the required areas of reinforcement at any point of a shell surface using the plastic design method. The thickness of

the shell was assumed to be given and the distribution of internal forces assumed to be known. Depending on the relative magnitudes of the bending moments, twisting moments, and the membrane forces, the optimal pattern of the reinforcement was chosen.

Selvanathan (1978) has considered the use of different number of columns and spacing of bracings to arrive at an optimal design of the Intze tank for 2 capacities using nonlinear programming. The cost of tank, staging and foundation has been chosen as the objective function and working stress method adopted for the design.

A method, which combines isoparametric finite element analysis with a nonlinear optimization procedure, to search for structural forms resulting in minimum cost reinforced concrete structures, has been proposed by Bond (1979). One of the examples considered is that of a conical water tank. The top diameter and thicknesses at top and bottom have been chosen as design variables.

The following studies do not make use of any optimization techniques in the strict sense, but consider ways of minimizing the cost of reinforced concrete tanks.

Gogate (1968) pointed out that the permissible steel stresses used in the design of tanks were highly conservative. Based on the research work on prediction of crack widths, a higher allowable stress was proposed. Wilby (1977b, 1978) has considered the cost minimization of polygonal tanks, in

stages. For a given capacity of the tank, the surface area is minimized in the first stage with a view to minimize the shuttering costs. Then the thickness of the rectangular plates and reinforcements in them are determined from individual yield-line collapse mechanisms.

Jain et al. (1979) have carried out extensive parametric studies to arrive at optimal design of Intze tanks. Various capacities, staging heights, bearing capacities of soils and lateral forces due to wind or earthquake are considered for the analysis. Based on these studies, a number of equations for rapid estimation of costs and materials have been presented.

1.3.3 Concluding remarks

It is obvious, from the foregoing literature review on the optimal design of reinforced concrete structures under consideration, that many important and relevant factors have been ignored. This might be due to limitation of either the methods adopted or scope of the study. The following factors need to be considered in order to widen the scope of application of these optimization studies.

In the design of slabs, proper estimation of dead load is very important because it is of the same order of magnitude as the imposed load. This will be possible only if the thickness of the slab is chosen as one of the design variables. Also, the cost of slab is very much dependent

on its thickness not only because of the cost of concrete but also because of the cost of secondary reinforcement which is proportional to the thickness of the slab.

A realistic study of optimal design of T-beams demands the inclusion of cost of shear reinforcement, side face reinforcement, and formwork.

All the optimization studies on reinforced concrete structures that have been carried out have essentially looked at the strength aspect only, serviceability requirements having been pushed into the background. The advent of ultimate strength design and high strength materials has resulted in more flexible structures for which serviceability considerations are as important as the strength requirement, if not more. A survey by Mayer and Rusch (1967) indicated that the most common cause of damage in reinforced concrete structures was excessive deflection. It may be pointed out that cracking and local damage, though not always serious, also need to be considered in the design.

Application of limit analysis to the design of water tanks has hardly been attempted. Optimal design of water tanks, with the exception of the Intze tank, has received very little attention although considerable amount of money is spent on the construction of water tanks. The complexity of the cracking problem, resulting in highly conservative allowable stresses for steel adopted for the design, might have been probably responsible for this.

The foregoing aspects of optimal design of reinforced concrete structures have been taken into account in the current study. The emphasis has been to obtain practically usable minimum cost designs, conforming to the codes of practice, that result in structures which will remain strong and serviceable throughout their design life.

1.4   Optimization

1.4.1   Optimum design

Design of any structure involves consideration of choice of materials, choice of design philosophy, cost or weight, construction methods and limitations in addition to other factors. Depending on the structure and circumstances, choice of some of these factors may be obvious. Then the designer has to arrive at a suitable form of the structure with all the required details. Any design philosophy would demand that the components of the structure be so designed that certain requirements are satisfied. Such requirements are termed as behaviour constraints in optimization literature. The construction or manufacturing processes may also impose certain restrictions on the design; these are termed as side constraints. Among the many designs of a structure which satisfy all the design constraints (feasible designs), some designs are better than the others. The criterion or criteria for evaluating a design, like cost or weight, is to

be first decided. This criterion, when expressed as a
function of design variables, is known as the merit function
or objective function. In the case of multi-criteria opti-
mization problem, the objective function is formulated with
different weightages for different criteria, the weightages
chosen reflecting the relative importance of the particular
criterion.

The usual method of indirect design, which does not
make use of any optimization technique, is a trial and error
process. An initial design is intuitively selected and
analysis carried out. If any design constraint is violated,
or margin of safety is too high the design variables are
suitably altered and analysis carried out again. This
procedure is repeated till a satisfactory feasible design is
obtained. If alternative feasible designs are considered,
then these designs are evaluated in terms of the chosen
criteria. Such a design procedure enables in arriving at a
design which is only better than the others.

On the other hand, the optimum seeking methods, also
known as mathematical programming techniques, start with an
initial design and proceed systematically to find the best
design which also satisfies all the constraints.

1.4.2   Optimization techniques

There are basically two approaches to optimization,
analytical and numerical. In optimal synthesis, the optimal

solution is determined directly in one step, usually employing an analytical method like variational calculus. This method, at present, is restricted to relatively simple optimization problems.

Linear programming, nonlinear programming, geometric programming, dynamic programming, quadratic programming, integer programming, game theory are some of the techniques in mathematical programming. Linear programming, which can be applied with comparative ease even for large systems, demands that the objective function as well as the constraints be linear functions of the design variables. Structural engineering designs do not usually satisfy this condition. Some of the other techniques like geometric programming and dynamic programming have been tried for structural design problems as reported earlier; but, the most widely used technique is the nonlinear programming which has been chosen for this study as well.

1.4.3   Nonlinear programming

Optimization is concerned with extremizing the value of objective function. Since the maximum of a function corresponds to the minimum of the negative of the same function, optimization can be stated as minimization of a function without any loss of generality.

Thus, the optimization problem can be stated as finding the vector $\bar{X}$ of the design variables which minimizes

the objective function $F(\overline{X})$, subject to the n constraints $g_j(\overline{X}) \leqslant 0$ (j = 1, 2,..., n). The methodology of most of the numerical methods of optimization is to produce a sequence of improved approximations to the optimum in the following manner:

$$\overline{X}_{i+1} = \overline{X}_i + \alpha_i^* \overline{s}_i , \qquad\qquad ..(1.1)$$

where $\overline{X}_{i+1}$ = vector of design variables at the new point,

$\overline{X}_i$ = vector of design variables at the previous point,

$\overline{s}_i$ = a suitable direction along which the value of objective function decreases, and

α_i^* = an appropriate step length for movement along $\overline{s}_i$.

The various methods to solve a nonlinear programming problem differ only in their approach to find $\overline{s}_i$ and α_i^*. The methods available to find the search direction $\overline{s}$ may broadly be classified as gradient methods which make use of the derivatives of the objective function in addition to objective function evaluation, and nongradient methods which need only objective function evaluation. Random search method, univariate method, pattern search methods are among the nongradient methods; steepest descent method, Newton's method, Fletcher-Reeves method, Davidon-Fletcher-Powell method are among the gradient methods. The methods to find α may be classified as elimination methods and interpolation

methods. Fibonaci method and golden section method among elimination methods, and quadratic interpolation and cubic interpolation method among interpolation methods are more popular.

All the methods mentioned so far are applicable to unconstrained optimization problems. But, engineering design deals essentially with constrained optimization problems. Certain modifications have to be incorporated in order that the methods mentioned earlier can be made applicable to this class of problems as well. It involves either transformation of the variables or use of penalty functions. Other than modified techniques of unconstrained optimization, direct methods, like method of feasible directions or constraint approximation method, are also available for the solution of constrained optimization problems.

1.4.4 Optimization methods used in the current study

The need for so many methods in nonlinear programming problems has arisen because of the inherent complexity in these problems. A method which works well for one problem may not be applicable for another problem. The efficiency of these methods is also problem-oriented.

Davidon-Fletcher-Powell method, also known as the variable metric method, has been found to be stable and applicable for a large class of problems. As such, this method has been chosen for the current study to find out

the search direction $\bar{S}$. Since this method makes use of gradient of the objective function, it has been coupled with the cubic interpolation method to find the appropriate step length α. Constraints have been taken care of through interior penalty function approach.

The constrained optimization problem is converted into an unconstrained optimization problem by constructing a function $\emptyset$ using the objective function $F(\bar{X})$, the n constraints $g_j(\bar{X})$, and a penalty parameter r as given by Eq. (1.2). If the unconstrained minimization of function $\emptyset$ is repeated for a sequence of values of the penalty parameter r_k (k = 1, 2,....), the solution may be brought to converge to that of the original problem. Hence, penalty function methods are also known as sequential unconstrained minimization techniques (SUMT). The function $\emptyset$ is defined in the form

$$\emptyset_k = \emptyset(\bar{X}, r_k) = F(\bar{X}) - r_k \sum_{j=1}^{n} \frac{1}{g_j(\bar{X})} \, . \qquad ..(1.2)$$

The penalty function is chosen such that its value will be small at points away from the constraint boundaries and will be very large as the constraint boundaries are approached. Therefore, if the algorithm starts with a feasible solution, all the subsequent solutions generated will also be within the feasible domain since the constraint boundaries act as barriers during the minimization process.

The reason for not carrying out the minimization of the $\emptyset$-function in only one stage using a small value of penalty parameter r, so that $\emptyset(\overline{X}, r)$ tends to $F(\overline{X})$, is that $\emptyset$ will be a very complex function away from the optimum and the algorithm will not get initiated if r is small. However, near the optimum point, the $\emptyset$-function is well behaved and permits computations to be carried with a small value of r. Therefore, minimization is carried out in stages with decreasing values of r, the initial design variable vector of any particular cycle being the final design variable vector of the previous cycle.

1.5   Need for Design Charts

Optimum design of structures with limited parametric studies achieves the following objectives:

(1)   response of the structure is better understood;

(2)   factors, which largely influence the cost or some other objective function, are identified;

(3)   need to modify the traditional notion on some of the design aspects gets highlighted; and

(4)   certain aspects of analysis and design which need further research are identified.

Thus, the incorporation of optimization concepts results in improved techniques.

In the field of civil engineering structural design, optimization has essentially remained a pursuit for research

(Goble and Moses, 1975). No doubt, it has achieved the above mentioned objectives. But, an average structural engineer is neither familiar with optimization methods nor can afford to pay for the necessary computer software and time to carry out optimization studies.

Therefore, for the structures considered in this study, extensive parametric studies have been made and design charts developed. In order to enable the design engineer to directly pick up the optimal design, optimization studies have been carried out within the framework of codes of practice. Indian standard specifications would have been the obvious choice. But, the Indian standard code of practice for the design of water retaining structures, based on the limit state approach, is not yet available. IS:456 (1978), though available for the limit state design of T-beam floors, has certain discrepancies (Subramanyam and Adidam, 1981a). Therefore, water tanks have been designed in accordance with BS 5337 (1976) and T-beam floors to conform to CP 110 (1970).

1.6   Organization of the Thesis

The thesis has been divided into seven chapters. Optimal limit state designs of T-beam floors, cylindrical surface water tanks without roof, those with roof, and overhead cylindrical water tanks with a flat bottom are dealt with in Chapters 2, 3, 4 and 5 respectively. The

general layout of each of these chapters has been, a brief discussion of the design considerations followed by the methods of analysis, formulation of the optimization problem, comparison of the results of optimal and indirect designs of an example, presentation of the design charts and discussion of the results of the parametric study. Chapter 6 deals with the computational aspects and stability of the optimum designs obtained. Summary and conclusions along with suggestions for future work are given in Chapter 7.

CHAPTER 2

OPTIMAL DESIGN OF T-BEAM FLOORS

2.1 General

Reinforced concrete beams very often carry a floor slab which is cast monolithically with it. It has been recognized for a long time that such beams can include part of the slab for structural action and hence a T-shaped section is considered in the analysis. In fact, even when a reinforced concrete slab is not monolithically cast with the beam, it is sometimes economical to adopt a T-section for the beam despite the additional shuttering costs. The advantage of the T-beam is its relatively wide horizontal flange of concrete which can provide the required compressive strength as well as increased lateral stability. The breadth of the flange which effectively acts with the rib and contributes to its flexural strength depends on the span of the beam, thickness of the slab, width of the rib and location of adjacent T- or L-beams. A combination of theory and practice has enabled the design rules to be formulated for both the flange breadth and the minimum amount of transverse reinforcement.

In many types of buildings, such as hospitals, hotels, hostels, colleges, offices, banks, shopping complexes, it is a common practice to provide a system of reinforced

concrete slabs and beams' in a single bay. This chapter deals with the optimum design of such T-beam floors.

Continuous one-way slabs resting on simply supported T-beams are considered. Mild conditions of exposure have been assumed to arrive at the cover for reinforcement.

2.2   Materials and Costs

In the limit state philosophy, the resistance of materials is expressed in terms of characteristic strengths, as outlined in Section 1.3.1.4, which takes into account the statistical variations in the resistance. CP 110 has chosen that value for the variability coefficient which implies that 5% of the test results will be below the characteristic value. Concrete is designated by a grade which immediately indicates its strength; thus, grade 20 concrete has a characteristic cube strength, at 28 days, of 20 MPa. In the case of reinforcement, the characteristic strength refers to either its yield stress or 0.2% proof stress.

To find the effect of strength of concrete on optimal designs, use of 20, 25 and 30 grades of concrete is examined. The additional strength and superior bond characteristics of deformed bars, at practically no extra cost, make them an obvious choice. Cold worked high yield strength deformed bars having a characteristic strength of 460 MPa for bars upto and including 16 mm diameter and 425 MPa for bars of higher diameter have been adopted. It is

presumed that in view of the saving in cost, the current trend of using mild steel links for shear reinforcement will make way for use of high strength deformed bars for shear reinforcement as well.

In order to compare the costs of designs using different grades of concrete, it is assumed that concrete of grades 25 and 30 will cost respectively 4% and 8% more than grade 20 concrete. This is based on the assessment of costs of the prescribed mixes given in CP 110, to produce concrete which has a typical range of slump of 25 mm to 75 mm using an aggregate with a nominal maximum size of 20 mm.

For reinforced concrete structures, not only the cost of materials, but the cost of formwork should also be included for a meaningful cost study. If cost optimization of structures is carried out using absolute costs of materials prevailing at the time of investigation, usefulness of the results will be limited when the costs of materials are different. Therefore, the practice of using cost ratio of materials is followed; costs of one m^3 of finished concrete and one m^2 of formwork have been normalized with cost of one newton of reinforcement and the resulting cost ratios termed R_1 and R_2 respectively.

2.3   Analysis

One-way continuous slabs are analysed on the basis of moment coefficients given in CP 110 for a three span

continuous beam. For strength computations, moment redis-
tributions of 0%, 10%, 20% and 30% are successively tried and
the one leading to the minimum value of objective function
automatically selected. For serviceability calculations,
redistribution of moments is not considered. Since the
imposed load is greater than the dead load for most of the
problems studied here, the reinforcement provided near the
top surface over the supports is extended up to midspan,
after curtailment, to take care of possible reversal of
bending moment. The beam is designed as a simply supported
T-beam.

2.4   Optimization

2.4.1   Design variables

The possible design variables are the thickness of
slab, areas of reinforcement at critical sections of the slab,
depth of beam, width of rib, areas of tension, compression,
shear and side face reinforcements for the beam. In simply
supported T-beams, there will be a large flange area to
resist compressive forces and hence there will normally be
no need for compression reinforcement. Preliminary studies
indicated that the proximity of compression reinforcement
to the neutral axis prohibits its effective use and in
optimum design, area of compression reinforcement should be
zero. Thus, the beam is designed as a singly reinforced

beam. When the width of rib, b_w, was considered as a design variable during the preliminary studies, it always assumed the minimum value prescribed. Therefore, it has been treated as a preassigned variable. Effect of rib width on the cost of beam has been discussed in Section 2.6.

Side face reinforcement has to be provided whenever the depth of beam exceeds 750 mm in order to control the width of cracks on the faces of the beam. Areas of side face reinforcement and shear reinforcement have not been explicitly treated as design variables, but their cost has been included in the objective function. The design variables considered are thus reduced to the following:

(1)   thickness of the slab, h_s,

(2)   depth of the beam, h_b,

(3)   area of tension reinforcement for the beam, A_{st},

(4)   area of tension reinforcement per unit width in the middle portion of end span of the slab, a_{s1},

(5)   area of tension reinforcement per unit width over the first interior support, a_{s2}, and

(6)   area of tension reinforcement per unit width over the middle portion of interior span, a_{s3}.

Details of the arrangement of reinforcement for the slab and beam are shown in Figures 2.1 and 2.2 respectively.
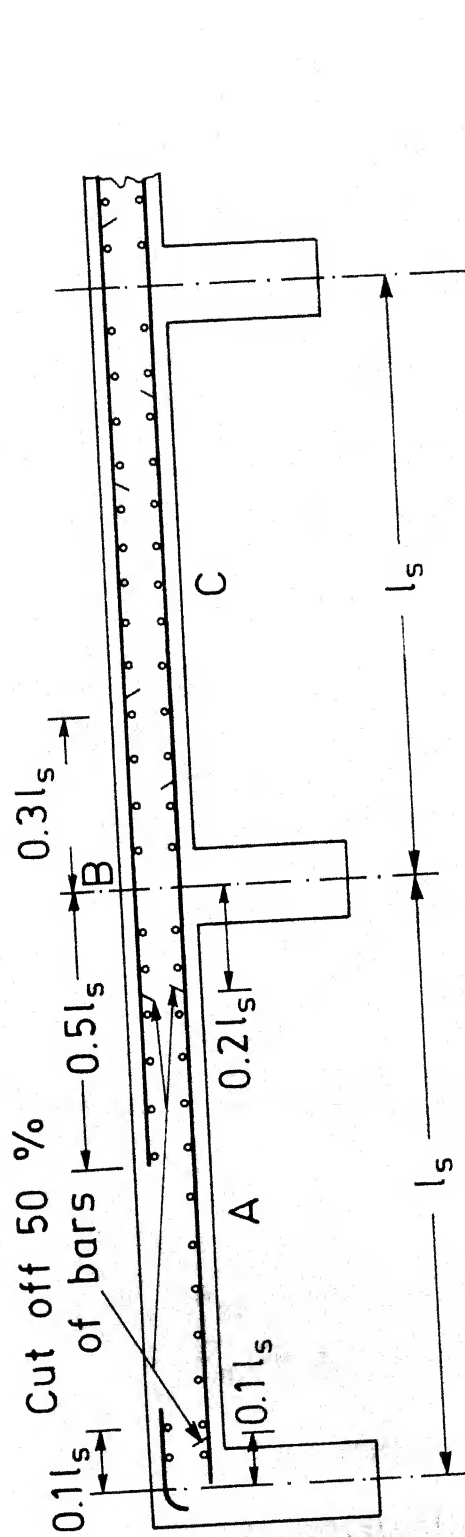
Fig. 2.1 - Details of slab reinforcement.
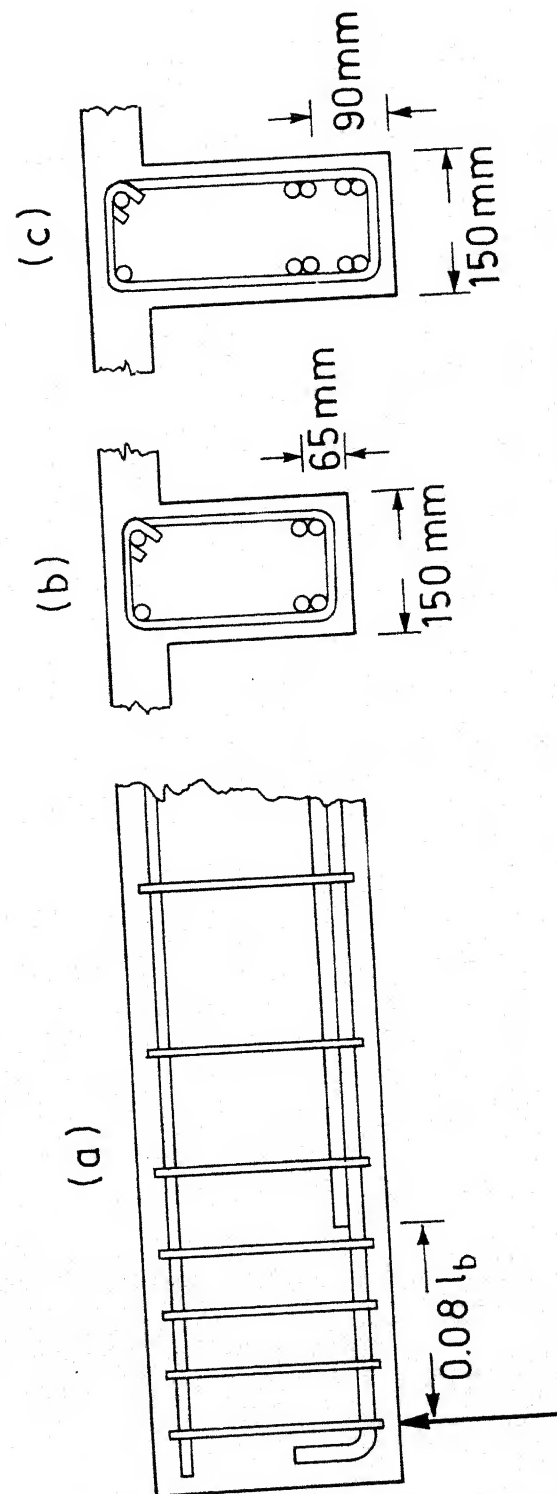
Fig. 2.2 - Details of beam reinforcement.

2.4.2   Objective function

The objective function, F, includes cost of concrete, cost of main and secondary reinforcements for the slab, tension, shear and side face reinforcements for the beam, and cost of formwork for the beam. Cost of formwork for the slab and hanger bars for the beam have not been included as these are constant and do not influence the optimal design. The width considered is from the centre of end span to the centre of interior span. The objective function, which represents the cost of the floor system per unit length of beam, can be written as

$$F = R_1\left[l_s h_s + (h_b - h_s)b_w\right] + \gamma_s\left[0.92\ A_{st} + A_{sv} + A_{sf}\right.$$

$$+ 0.4\ l_s(a_{s1} + 2a_{s2} + a_{s3}) + 0.0012\ l_s h_s\left.\right]$$

$$+ R_2\left[2(h_b - h_s) + b_w\right], \qquad\qquad ..(2.1)$$

where l_s = span of the slab,

A_{sv} = average area of shear reinforcement per unit
length of beam,

A_{sf} = area of side face reinforcement, and

γ_s = specific weight of steel.

2.4.3   Constraints

The constraints must ensure that the T-beam floor has the required safety factor against attaining the various limit states. The limit states considered here are the

ultimate limit state of flexure and serviceability limit state of deflection for the slab, ultimate limit states of flexure, shear, and bond besides serviceability limit states of deflection and cracking for the beam. Shear and bond are not critical for design of slabs considered here. The reason for excluding the limit state of cracking for the design of slabs is explained in the next paragraph. Some more aspects of structural cracking have been discussed later in Section 3.1.4.

CP 110 has given the following equation for determination of w_{cr}, estimated width of surface flexural cracks which should not exceed 0.3 mm for mild conditions of exposure:

$$w_{cr} = \frac{3\,a_{cr}\,\varepsilon_m}{1 + 2\,\dfrac{(a_{cr} - c_{min})}{(h - d_n)}} \, , \qquad\qquad ..(2.2)$$

where $\varepsilon_m = \dfrac{\varepsilon_s y}{d - d_n} - \dfrac{1.2\,b_t hy}{A_s(h - d_n)f_y} \times 10^{-3}$, $\qquad ..(2.3)$

a_{cr} = distance from the point under consideration to the surface of the nearest longitudinal bar,

c_{min} = minimum cover to the tension reinforcement,

h = depth of the member,

d_n = depth of neutral axis,

ε_s = strain in steel at service loads,

y = distance from neutral axis to level of point considered,

d   = effective depth of the member,

b_t = width of the member at centroid of tension reinforcement, and

f_y = characteristic strength of reinforcement.

The second term in Eq. (2.3) represents the stiffening effect of concrete in the tension zone and where the steel percentage is small, this effect can be very significant. This is reflected in the provisions of the code by allowing a maximum spacing of 5d when the steel percentage is less than 0.5%. It has been found that in the optimum design of slabs considered in this study, the maximum steel percentage at sections where it is fully stressed is 0.3. Using an average partial safety factor of 1.5 for loads and a value of 460 MPa for f_y, value of $\varepsilon_s = 1.333 \times 10^{-3}$. The optimum thickness of slabs is found to vary between 80 mm and 160 mm for the range of slabs considered in this study. Thus, estimating the maximum value of $\dfrac{y}{d - d_n} = \dfrac{h - d_n}{d - d_n}$ as 1.75 to 1.25 from the thinner slab to the thicker slab, for a steel percentage of 0.3, the value of ε_m will be in the range 1.46×10^{-3} to 0.8×10^{-3}. At one extreme, for points directly below the reinforcement, $a_{cr} = c_{min}$ which can be taken as 30 mm. For such points, the crack width given by Eq. (2.2) will be in the range 0.132 mm to 0.072 mm, from the thinner slab to the thicker slab. At the other extreme, for large values of a_{cr}, an upper bound on the value of w_{cr} is $1.5(h - d_n) \varepsilon_m$. For the estimated values of ε_m, the crack width at such

points will be about 0.16 mm to 0.18 mm. Therefore, when the conditions of exposure are mild, structural cracking is not a problem for slabs which are neither thick nor have a high percentage of reinforcement.

The first three constraints make sure that m_{uA}, m_{uB} and m_{uC} the moments of resistance per unit width of slab at sections A, B and C (Figure 2.1), are not less than M_A, M_B and M_C, the bending moments per unit width due to ultimate loads at these points. The values of M_A, M_B and M_C are arrived at after considering the redistribution of moments. It is necessary that all the constraints are expressed in a normalized form to avoid numerical complications. Thus, the first three constraints can be stated as,

$$\frac{M_A}{m_{uA}} - 1 \leq 0, \qquad ..(2.4)$$

$$\frac{M_B}{m_{uB}} - 1 \leq 0, \qquad ..(2.5)$$

$$\text{and} \qquad \frac{M_C}{m_{uC}} - 1 \leq 0. \qquad ..(2.6)$$

Point A is critical from the deflection point of view. In order that the deflections of the slab are within permissible limits, the ratio of span l_s to effective depth d_s should not exceed 26 ξ for a continuous slab as per CP 110, where ξ is a modification factor whose value depends on the stress in the reinforcement at service loads and

percentage area of tension reinforcement both considered at
point A. Thus, the fourth constraint can be written in the
form

$$\frac{l_s}{26 \xi \; d_s} - 1 \leq 0 . \qquad\qquad ..(2.7)$$

The other constraints in the design of slab pertain
to the minimum percentages of reinforcement to be provided
at different locations as required by CP 110. These are not
side constraints as the specified minimum areas have to be
provided for the satisfactory behaviour of the floor.

For effective T-beam action, the minimum area of
reinforcement near the top surface of the slab for the full
effective width of flange is 0.3% of the gross area of longi-
tudinal section of the flange. This in effect is a lower
bound on a_{s2}. The minimum area of main reinforcement is
0.15%. Constraints (5) to (7) can be stated as

$$\frac{0.003 \; h_s}{a_{s2}} - 1 \leq 0 , \qquad\qquad ..(2.8)$$

$$\frac{0.0015 \; d_s}{a_{s1}} - 1 \leq 0 , \qquad\qquad ..(2.9)$$

and $$\frac{0.0015 \; d_s}{a_{s3}} - 1 \leq 0 . \qquad\qquad ..(2.10)$$

The eighth constraint ensures that the moment of
resistance M_u of the beam is not less than M, the maximum
bending moment due to ultimate loads. Thus,

$$\frac{M}{M_u} - 1 \leq 0 \ . \qquad\qquad ..(2.11)$$

The ninth and tenth constraints deal with the shear and bond capacities of the beam respectively. The shear capacity V_u of the beam should not be less than V, the maximum shear force due to ultimate loads. The maximum bond stress f_{bs}, even after curtailment of reinforcement, should not be greater than the permissible value f_{dbs}. These constraints can be written in the form

$$\frac{V}{V_u} - 1 \leq 0 \ , \qquad\qquad ..(2.12)$$

and
$$\frac{f_{bs}}{f_{dbs}} - 1 \leq 0 \ . \qquad\qquad ..(2.13)$$

The minimum area of shear reinforcement as well as the additional reinforcement required near the supports is computed and included in the objective function. The minimum area of tension reinforcement for the beam is 0.15% of $b_w d_b$, where d_b is the effective depth of beam. The eleventh constraint is stated as

$$\frac{0.0015 \ b_w d_b}{A_{st}} - 1 \leq 0 \ . \qquad\qquad ..(2.14)$$

Detailed computations have been carried out to find w_{cr}, the maximum surface crack width in the beam and y_{max}, the maximum long term deflection in the beam including effects of creep and shrinkage. The twelfth and thirteenth

constraints make sure that these values do not exceed the
permissible values. Thus,

$$\frac{w_{cr}}{0.3} - 1 \leqslant 0,\qquad\qquad ..(2.15)$$

and
$$\frac{250\ y_{max}}{l_b} - 1 \leqslant 0,\qquad\qquad ..(2.16)$$

where l_b is the span of the beam.

2.5   Example

Optimum design of a typical T-beam floor is consi-
dered choosing the design variables, objective function, and
constraints discussed in Section 2.4. The floor consists of
a three-span continuous slab of span 3500 mm resting on
simply supported T-beams of span 8750 mm and subjected to an
imposed load $q_k = 4$ kN/m^2. Concrete of grade 25 has been
adopted. The results are compared with that of an efficient
indirect design, details of which are as follows:

Assuming h_s as 125 mm and using moment coefficients
with 10% redistribution of moments as given in CP 110, values
of moments due to ultimate loads at locations A, B and C
(Figure 2.1) in kNm/m are 11.46, 12.87 and 7.13 respectively.
The bending moment due to service loads at section A is
7.9 kNm/m. Areas of reinforcement provided at A, B and C
in mm^2/m are 310, 375 and 185 respectively. With an effective
depth of 100 mm, the moments of resistance at these sections

in kNm/m are 11.53, 13.99 and 7.15 respectively and hence just satisfactory at A and C. At B, since the area of rein- forcement provided corresponds to the minimum required as per CP 110 (0.3%), it cannot be reduced any further. The stress in reinforcement due to service loads at section A is 281 MPa which leads to an allowable span to effective depth ratio of 36. Since the actual span to effective depth ratio is 35, the thickness of slab cannot be decreased.

The beam is designed on the basis of an overall depth of 550 mm and a rib width of 225 mm. The maximum bending moment and shear force in the beam due to ultimate loads are 386 kNm and 177 kN respectively. An area of tension reinforcement equal to 2250 mm^2 and provided at an effective depth of 485 mm results in a moment of resistance of 387 kNm. The shear reinforcement provided consists of 8 mm diameter 2 legged stirrups at 170 mm centre to centre for the first 1.87 m from the supports; the spacing is doubled for the remaining length. Behaviour of the beam in respect of deflection and cracking has been found to be satisfactory.

Although it is obvious that the design variables associated with the slab are independent of cost ratio R_2, optimization studies have revealed that they are practically independent of cost ratio R_1 as well. Optimal values of h_s, a_{s1}, a_{s2} and a_{s3} obtained for all the cost ratios have been found to be around 112.5 mm, 560 mm^2/m, 340 mm^2/m and

$210 \text{ mm}^2/\text{m}$ respectively. The Table 2.1 shows the values of other design variables for the optimal design and the values of objective function for optimal designs (OPD) and indirect design (IND) for different cost ratio combinations. It is observed that the value of b_w has always assumed the minimum value prescribed and optimal depth of beam decreases with an increase in the values of both R_1 and R_2 with a consequent increase in the value of tension reinforcement. It is also observed that the value of design variables have not changed with an increase in the value of R_2 for $R_1 = 1872$. In the normal course, the beam depth should have decreased and area of reinforcement increased. But, when the required value of A_{st} is more than 1963 mm^2 (4 bars of 25 mm), values of both cover and width of rib increase and the objective function at this point has a stepwise increase.

In this example, savings in the cost due to optimization averages to about 11%. This value is obviously linked to the other design with which the comparison is made. When the value of objective function for a textbook solution (Jaikrishna, 1971) was compared with those of optimum designs, it was observed that the savings in cost was about 16% when mild steel reinforcement was adopted, and increased to 22% when the reinforcement was changed to high yield strength deformed bars (Subramanyam and Adidam, 1980). In the present example, if a smaller value had been assumed for the width of rib, savings due to optimization would have reduced by 1% to 2%.

Table 2.1   Comparison of optimal and indirect designs

Cost ratio R_1	Cost ratio R_2	Optimal beam dimensions			Objective function	
		b_w (mm)	h_b (mm)	A_{st} (mm^2)	OPD	IND
624	20	150	750	1480	668	758
	50	150	650	1720	708	790
	80	150	610	1845	743	822
1248	20	150	635	1762	967	1091
	50	150	580	1945	1001	1123
	80	150	575	1962	1034	1155
1872	20	150	575	1962	1258	1424
	50	150	575	1962	1291	1456
	80	150	575	1962	1323	1488

2.6   Effect of Rib Width

Optimal synthesis indicates that a beam of very narrow width and large depth will be optimal, when only resistance to bending is considered. On the other hand, it is generally considered a good design practice to provide a fairly wide rib to resist the shear force.

Since the preliminary studies indicated that the width of rib always assumed the minimum value prescribed, the question of what this minimum should be merits detailed consideration. Functionally, the rib should be deep enough to provide the required lever arm from flexural, shear and bond strength considerations and wide enough to place the reinforcement suitably. While the depth is easily fixed from strength considerations, the width of rib is arbitrarily chosen. A wide rib reduces the shear stress and can accommodate more number of reinforcing bars per layer thereby increasing the effective depth. But, the quantity of concrete required, amount of minimum shear reinforcement to be provided and the perimeter of shear reinforcement link, all get increased.

In order to find out the effect of width of rib on the optimal solution, separate studies were carried out on three different problems for different cost ratio combinations. Part of the result of these studies is given in Table 2.2. These studies indicate that, for the cost to be minimum, a beam with as narrow a rib as possible from

Table 2.2 Effect of variation of rib width on optimum design

$$R_1 = 1200, \quad R_2 = 50$$

Problem No.	Data	b_w (mm)	h_b (mm)	A_{st} (mm^2)	Effective cover (mm)	Objective function	Increase in cost (%)
1	$l_s = 3.5$ m	130	490	975	45	225	0
	$l_b = 7.0$ m	180	380	1420 (1390)*	65 (65)	250 (228)	12 (2)
	$q_k = 3.0$ kN/m^2	230	330	1580 (1975)	45 (65)	268 (238)	19 (6)
2	$l_s = 4.0$ m	160	605	2040	65	360	0
	$l_b = 9.0$ m	230	535	2410 (2350)	65 (65)	410 (365)	14 (1)
	$q_k = 3.0$ kN/m^2	250	495	2560 (2575)	50 (65)	416 (372)	15 (3)
3	$l_s = 5.0$ m	160	750	3215	65	511	0
	$l_b = 10.0$ m	225	745	3305 (3370)	65 (90)	561 (527)	10 (3)
	$q_k = 4.0$ kN/m^2	275	685	3605 (3725)	55 (90)	597 (539)	17 (6)

* Quantities within brackets indicate values corresponding to beam with the narrowest rib of the respective problem, designed to have the same depth as the optimum depth of beam with a wider rib.

practical considerations should be used. It is observed that, for the same problem, optimal depth of a beam with a narrow rib is more compared with that of a wider rib. If the depth of beam is to be restricted, a beam with a narrow rib could still be advantageously used with a lesser depth than the optimal. Quantities like cover, area of reinforcement etc., pertaining to the suboptimal design of a beam with a narrow rib, assigned optimal depths of beams with wider ribs, are indicated, within brackets, in Table 2.2.

Large depths and low reinforcement ratios are characteristic of an optimal design of a beam. For such a design, the neutral axis of a T-beam will invariably be in the flange and hence the width of rib does not affect the stability consideration of T-beams. When the area of reinforcement required does not exceed 980 mm^2 (2 bars of 25 mm), it is possible to have a rib width of 130 mm without violating the cover and minimum spacing requirements of the code. But, the width of rib adopted in the current study is 150 mm so that there is no congestion of reinforcement. This is increased to 160 mm when 32 mm bars are required to be used.

When grade 25 concrete is used, effective cover provided to the centroid of reinforcement is 40 mm, 55 mm, 65 mm and 90 mm, when the area of tension reinforcement is not greater than the area of 2 bars of 25 mm, 4 bars of 25 mm, 4 bars of 32 mm and 8 bars of 25 mm respectively. When 8 bars are to be provided, they may be grouped as

shown in Figure 2.2. For other grades of concrete, values of effective cover will be marginally different.

2.7   Parametric Studies

Optimal limit state design of T-beam floors has been carried out for the following cases:

(1)   grades of concrete :   20, 25 and 30;

(2)   spans of slab in mm :   2500, 3000, 3500, 4000, 4500 and 5000;

(3)   ratios of span of beam to span of slab :   2.0, 2.25 and 2.5;

(4)   intensities of imposed load in kN/m^2 :   2.0, 3.0 and 4.0;

(5)   cost ratios R_1 :   600, 1200 and 1800; and

(6)   cost ratios R_2 :   20, 50 and 80.

Cost ratio R_1 has been taken as 624, 1248, and 1872 for grade 25 concrete in view of the estimated 4% increase in the cost of grade 25 concrete in comparison with grade 20 concrete. Similarly, R_1 has been taken as 648, 1296, and 1944 for grade 30 concrete. This will enable one to compare the cost of the floor system designed with different mixes of concrete. The middle values in all these cost ratios represent the prevailing costs in India at present.

2.8   Results and Discussion

The observation from the example considered in Section 2.5 that the values of design variables associated with slab are almost independent of R_1 has been found to be true for all cases. In general, it is expected that other factors remaining same, slabs would be thicker for a low value of R_1 (cost of concrete relatively less) and thinner for a high value of R_1 (cost of concrete relatively more). Values of a_{s1} and a_{s3} no doubt decrease with an increase in slab thickness; but, values of both secondary reinforcement and a_{s2}, which are linked to the slab thickness, increase along with it.

Although slabs tend to be marginally thicker for cost ratio $R_1 = 600$ and marginally thinner for cost ratio $R_1 = 1800$, the difference is so small that it can be neglected, especially when it is recognized that the slab thickness will be rounded off to the next 5 mm value. Therefore, only one set of design charts has been prepared to find the optimal values of design variables h_s, a_{s1} and a_{s3}. These are shown in Figures 2.3 through 2.5. These correspond to grade 25 concrete. Optimal thickness of slabs for grades 20 and 30 will be 5 mm to 6 mm more and less respectively as can be seen from Tables 2.3 through 2.5.

In all the optimal designs, it has been observed that the constraint regarding the minimum value of a_{s2} is critical. Because of the high characteristic strength of

Table 2.3 Effect of strength of concrete on the optimal design

$l_s = 4000$ mm, $l_b = 8000$ mm, $q_k = 2$ kN/m², $R_2 = 80$

R_1	Concrete grade	h_s (mm)	a_{s1} (mm²/m)	a_{s2} (mm²/m)	a_{s3} (mm²/m)	h_b (mm)	A_{st} (mm²)	Objective function
600	20	121	520	365	165	515	1535	760
	25	115	520	345	160	495	1540	746
	30	109	530	330	160	475	1540	733
1200	20	120	555	360	170	470	1720	1083
	25	115	520	345	160	450	1725	1066
	30	108	560	325	165	430	1720	1045
1800	20	119	600	360	180	440	1875	1401
	25	114	560	345	175	420	1875	1380
	30	106	650	320	185	400	1870	1358

Table 2.5   Effect of strength of concrete on the optimal design

l_s = 5000 mm, l_b = 11,250 mm, q_k = 4 kN/m^2, R_2 = 50

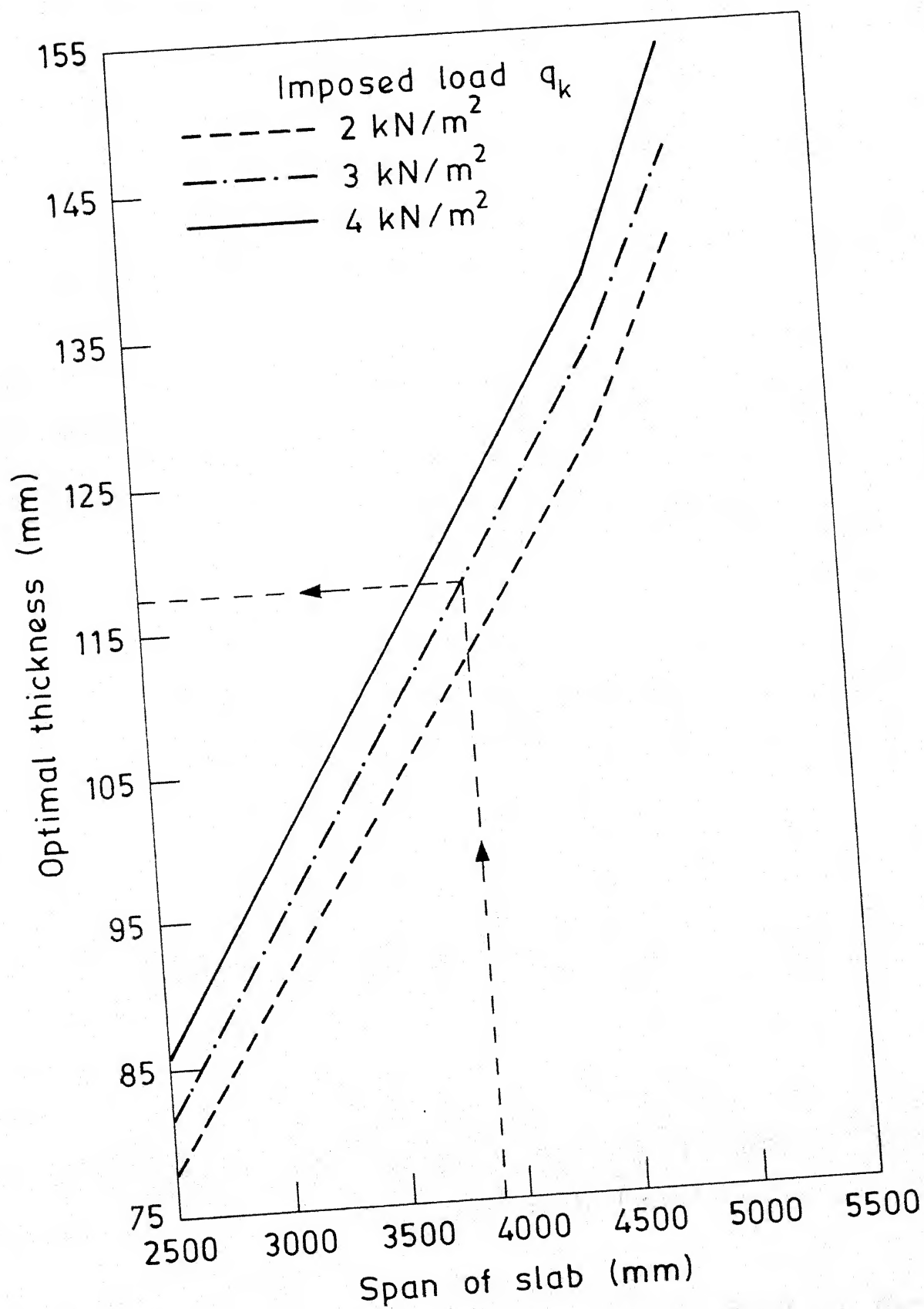| R_1	Concrete grade	h_s (mm)	a_{s1} (mm^2/m)	a_{s2} (mm^2/m)	a_{s3} (mm^2/m)	h_b (mm)	A_{st} (mm^2)	Objective function
600	20	160	865	480	380	1005	2980	1387
	25	153	920	460	375	990	2960	1375
	30	150	790	450	350	980	2940	1360
1200	20	160	865	480	380	940	3200	1945
	25	153	920	460	375	935	3220	1932
	30	149	840	450	380	915	3220	1923
1800	20	158	975	475	435	900	3480	1997
	25	151	1000	455	425	870	3480	1982
	30	148	900	445	400	860	3460	1972

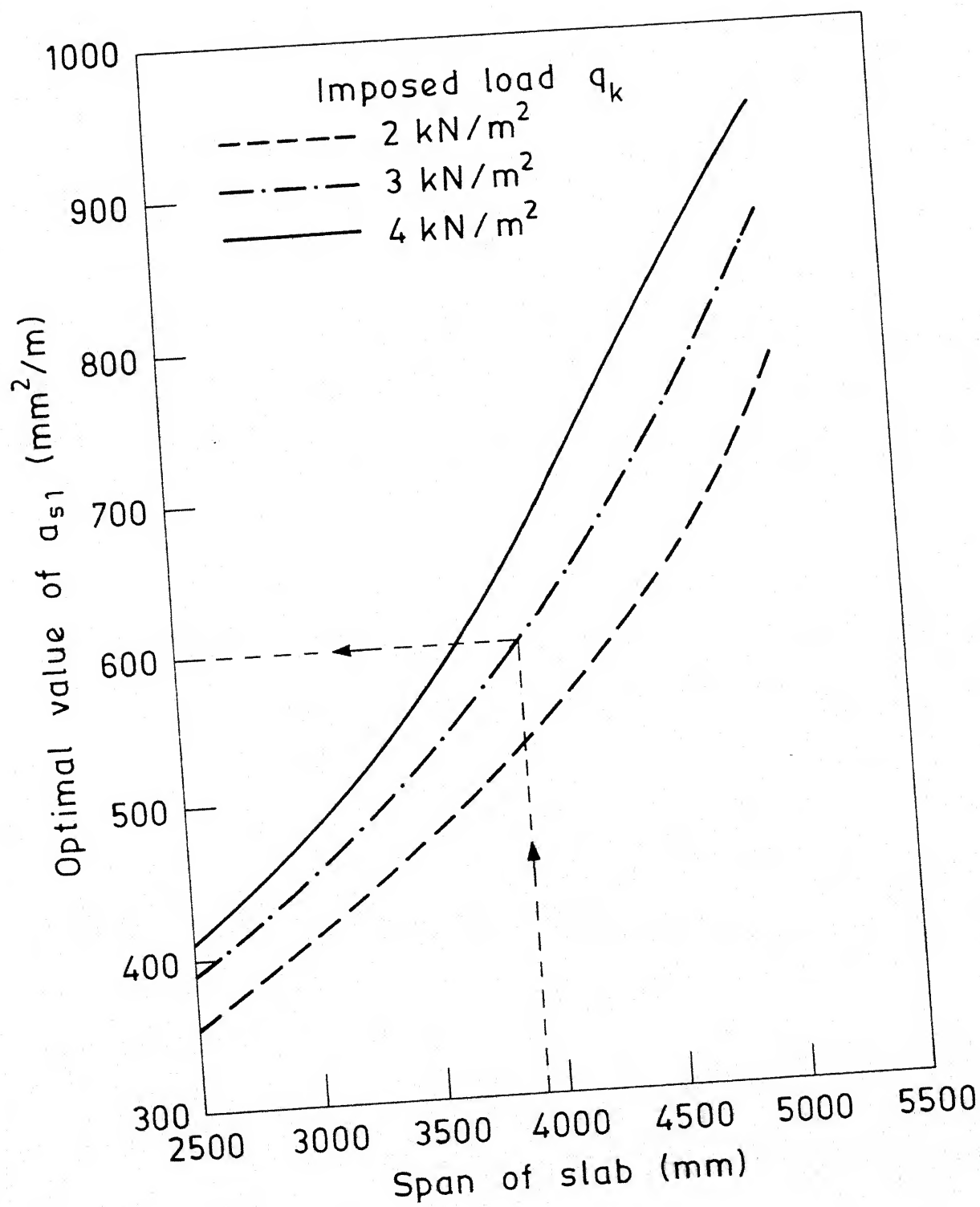Fig. 2.3 - Design chart for optimal thickness of slab.

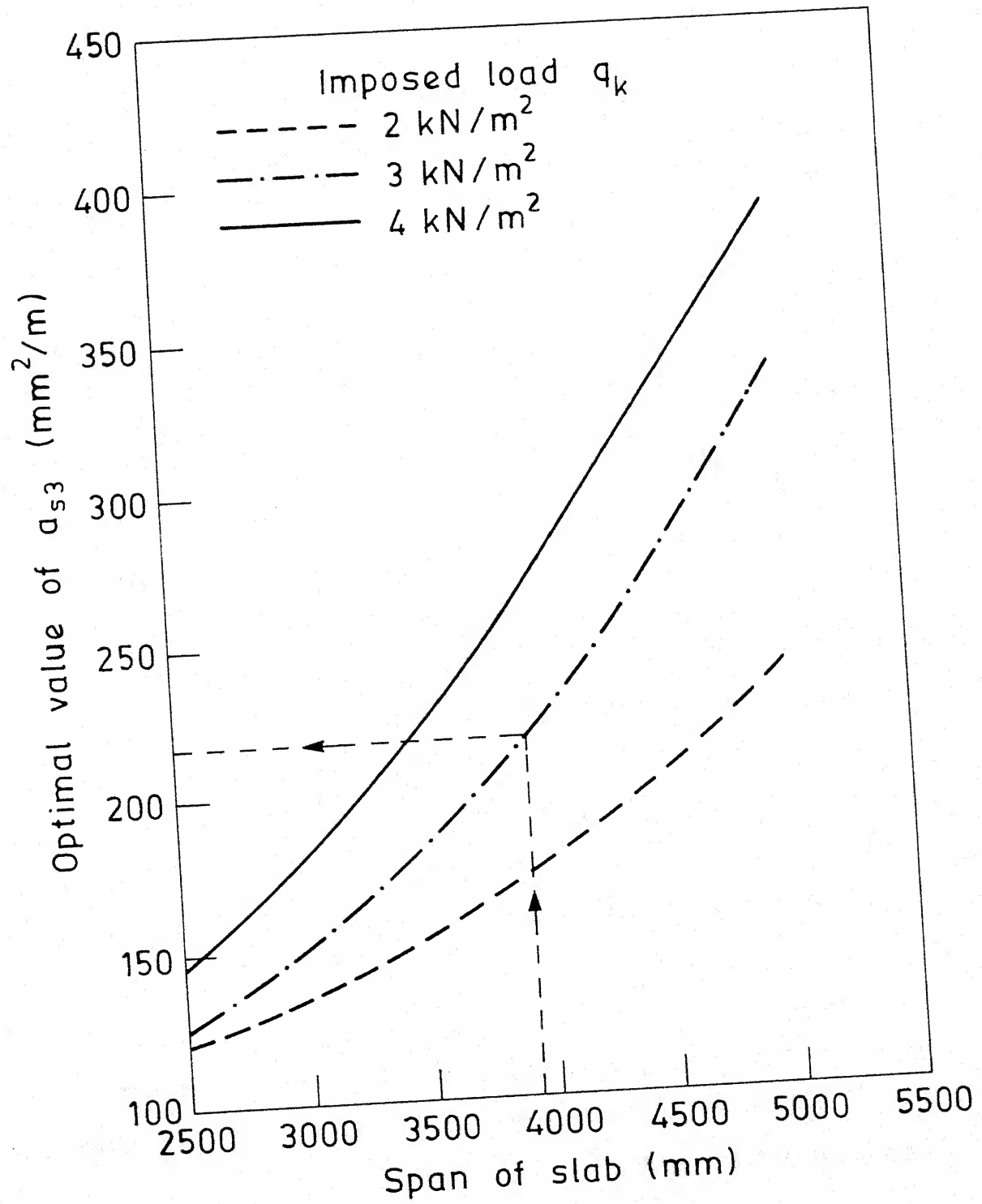Fig. 2.4 - Design chart for optimal value of a_{s1}.

Fig. 2.5 - Design chart for optimal value of a_{s3}.

the reinforcement (460 MPa) and advantage of redistribution of moments, a value of 0.3% is not required for a_{s2} from the strength point of view, except for large spans and heavy loads. When the designs were carried out as per IS:456-1978, which does not insist on such a minimum, even with a characteristic strength of 415 MPa, optimal value for a_{s2} was found to be slightly less than 0.3% (Subramanyam and Adidam, 1981b). Therefore the requirement of CP 110 regarding a minimum area of 0.3% of gross area of flange for its full effective width needs careful consideration. If it is established that a lower value of a_{s2} is permissible when the characteristic strength of reinforcement is high, it would enable in reducing the cost of T-beams further.

The value of a_{s1} is controlled by deflection considerations. Since the reinforcement considerations discussed earlier demand a thin slab, it is found that the value of a_{s1} is comparatively high (0.45% to 0.75%) so that stresses at service loads could be low. This results in a higher value of modification factor ξ and hence permits a lower thickness.

Thus, it will be advantageous to provide more reinforcement in the end span than is required from the strength point of view. Such a conclusion had in fact been reached, even when a single design variable had been adopted to specify the reinforcement in slab (a_{s2}) and other values (a_{s1} and a_{s3}) were not independent but simply proportional

to a_{s2}, which meant that at all sections the reinforcement provided was more than that required from strength considerations (Subramanyam and Adidam, 1981c).

Around the optimum solution, value of a_{s1} is very sensitive to the thickness and a small reduction in the value of h_s will demand a large increase in the value of a_{s1}. Therefore, if a higher value than the optimum is chosen for the thickness of slab, either due to rounding off to the nearest convenient value or due to some other considerations, it is better to evaluate the value of a_{s1} for the chosen thickness as it will be considerably less than the value associated with the optimum thickness. Value of a_{s3} is governed entirely from strength considerations and it is sensitive to changes in thickness only for large spans.

Optimal values of h_b and A_{st} are again governed essentially from strength considerations. For any chosen problem and cost ratio combination, it is found that the value of objective function is essentially same for the three grades of concrete considered, though it marginally decreases with an increase in strength. This is illustrated in Tables 2.3 through 2.5. Another important observation is that there is no significant change in the value of A_{st} for the three grades of concrete, other factors remaining same, though the value of h_b increases with decrease in the strength of concrete.

Since a number of parameters were considered for the parametric studies, the generated volume of results is quite considerable. In order to restrict the number of tables or charts, the following procedure has been adopted:

(1) The maximum bending moment in the beam due to ultimate loads, M, has been computed for the various cases considered and the design variables h_b and A_{st} related to the value of M through design charts. This has been done for different cost ratio combinations.

(2) Since A_{st} has been found to be essentially same for different grades of concrete, other factors remaining same, design charts to find optimal values of h_b and A_{st} have been prepared for grade 25 concrete only. These are given in Figures 2.6 through 2.11. For other grades of concrete, knowing the optimal value of A_{st} from Figures 2.9 through 2.11, corresponding value of h_b can be computed from the bending moment considerations.

It is seen from Figures 2.6 through 2.8 that the beam depth stays constant at 750 mm for a particular range of values of M. This is because of the code stipulation that, in order to control crack widths on the sides of the beam, side face reinforcement has to be provided for two-thirds the depth of beam when the beam depth exceeds 750 mm. This causes a sudden jump in the value of objective function, which acts as a sort of barrier. So, the depth of beam stays
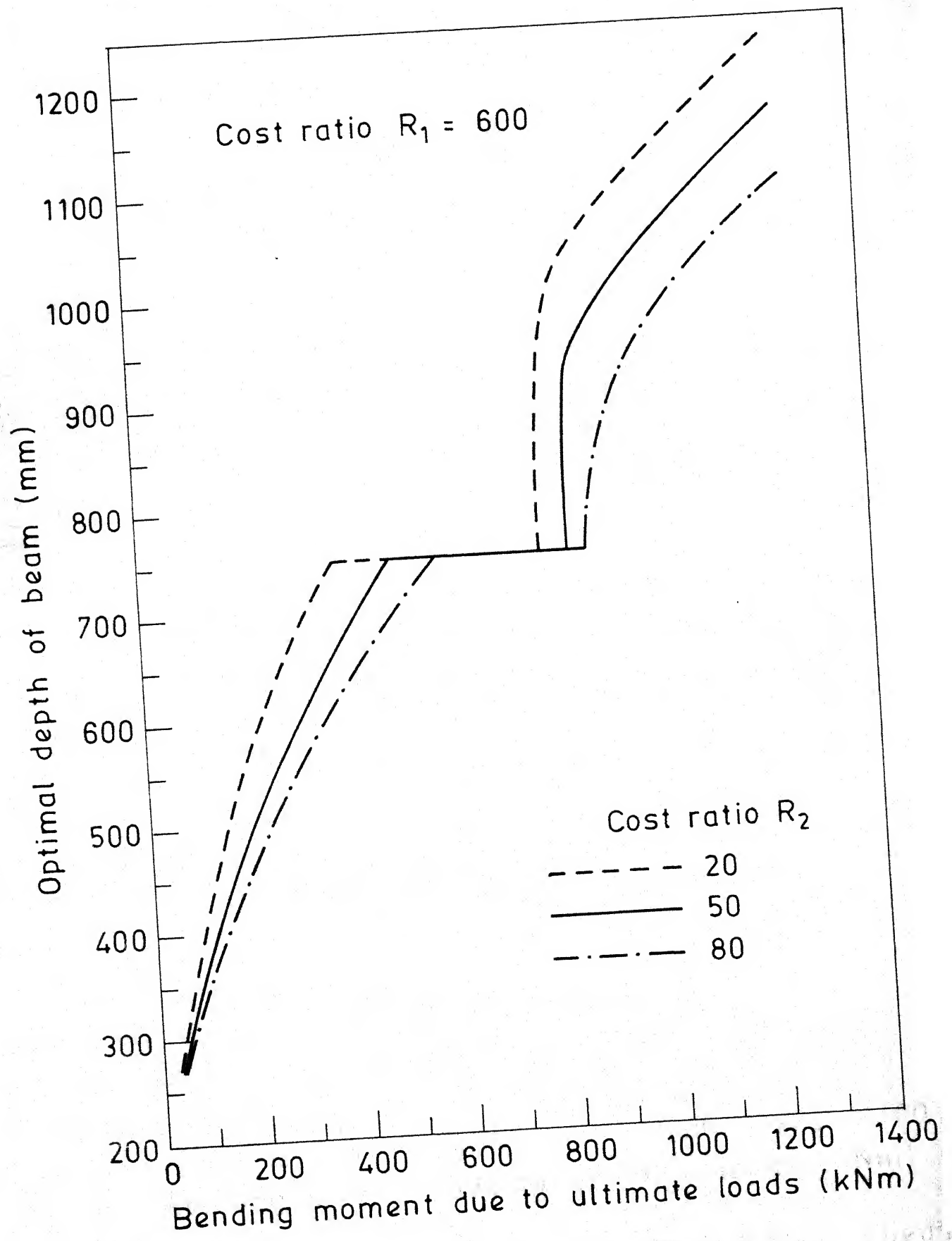
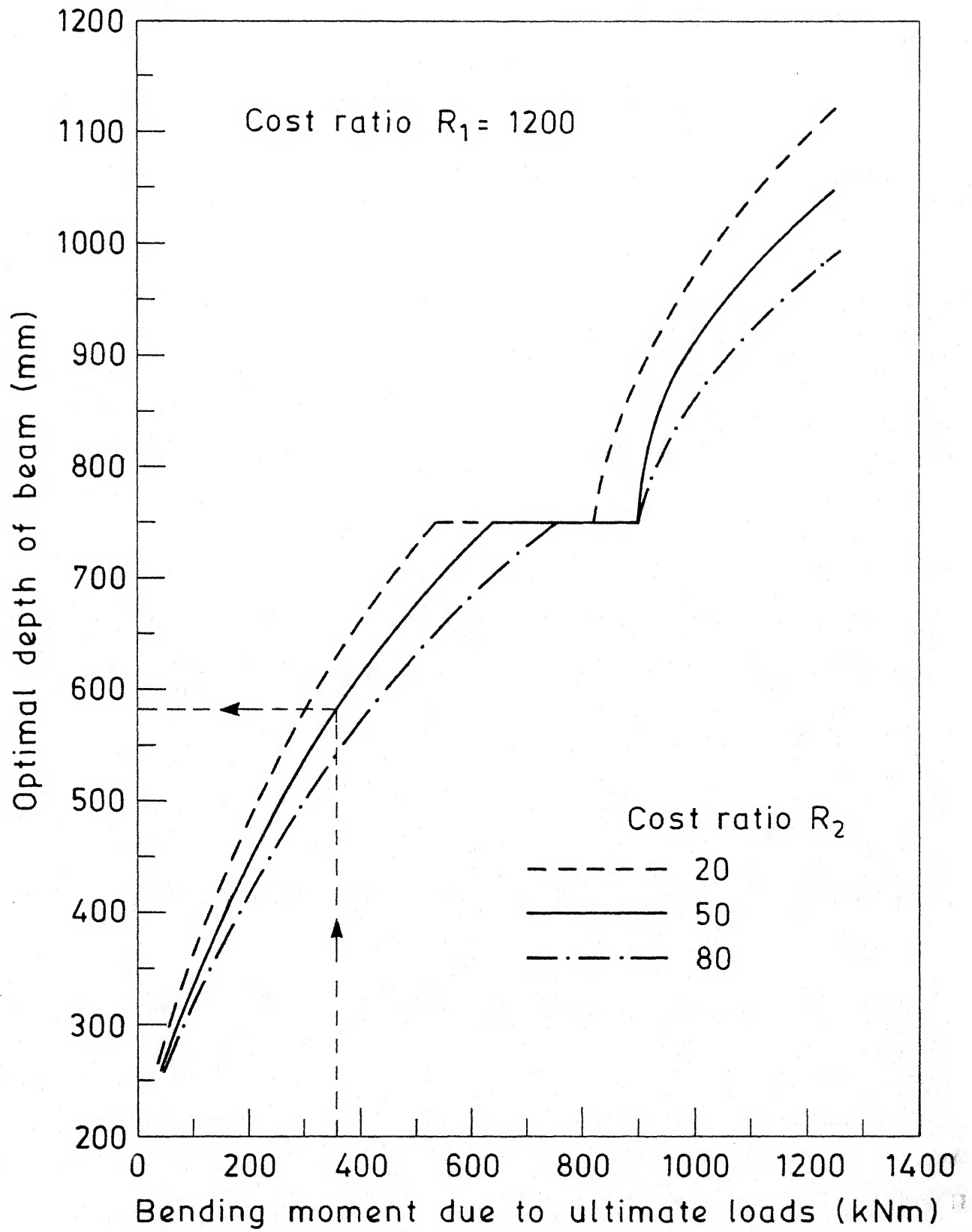Fig. 2.6- Design chart for optimal depth of beam.
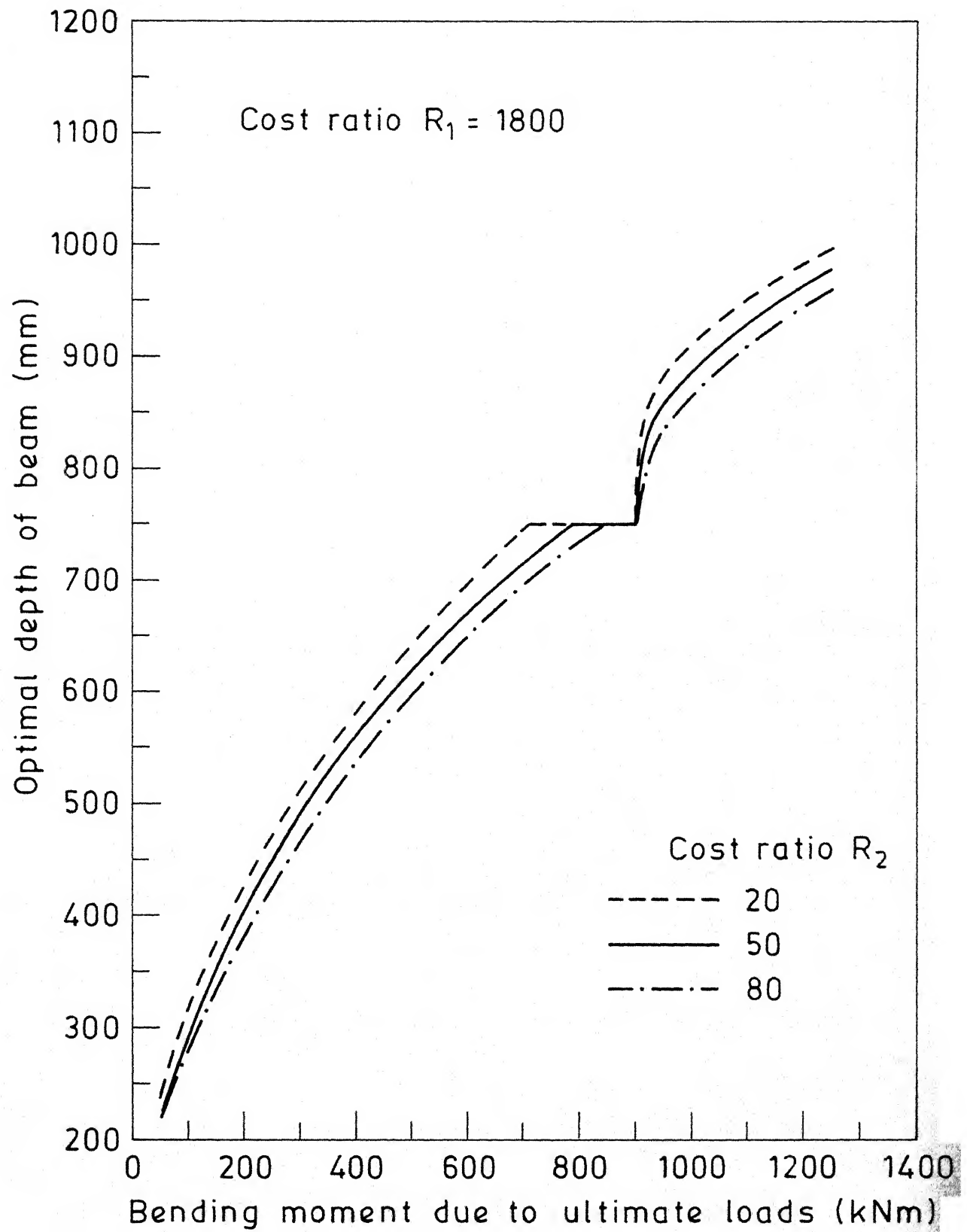
Fig. 2.7 - Design chart for optimal depth of beam

Fig. 2.8 - Design chart for optimal depth of beam.
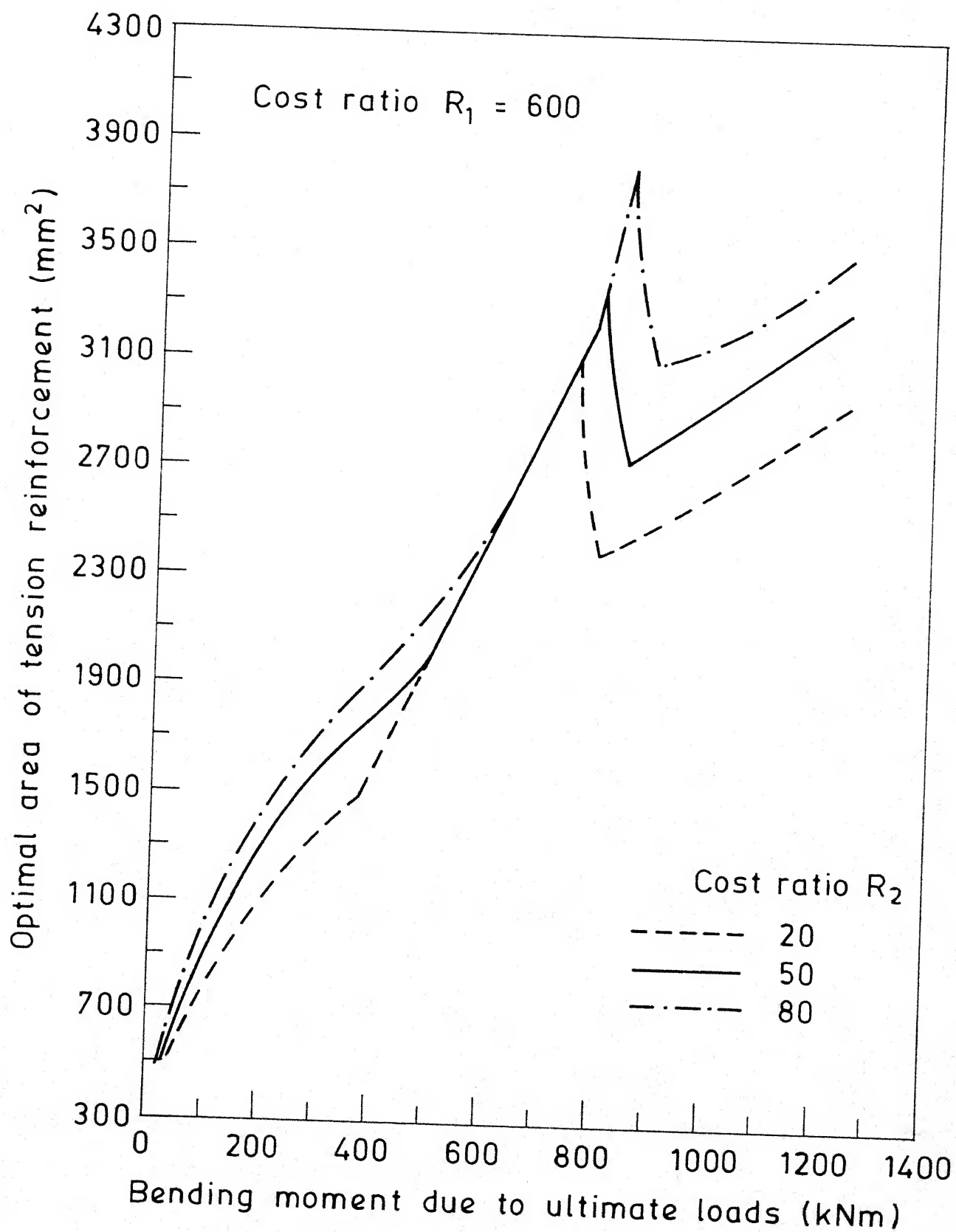
Fig. 2.9 - Design chart for optimal area of tension reinforcement for beam.
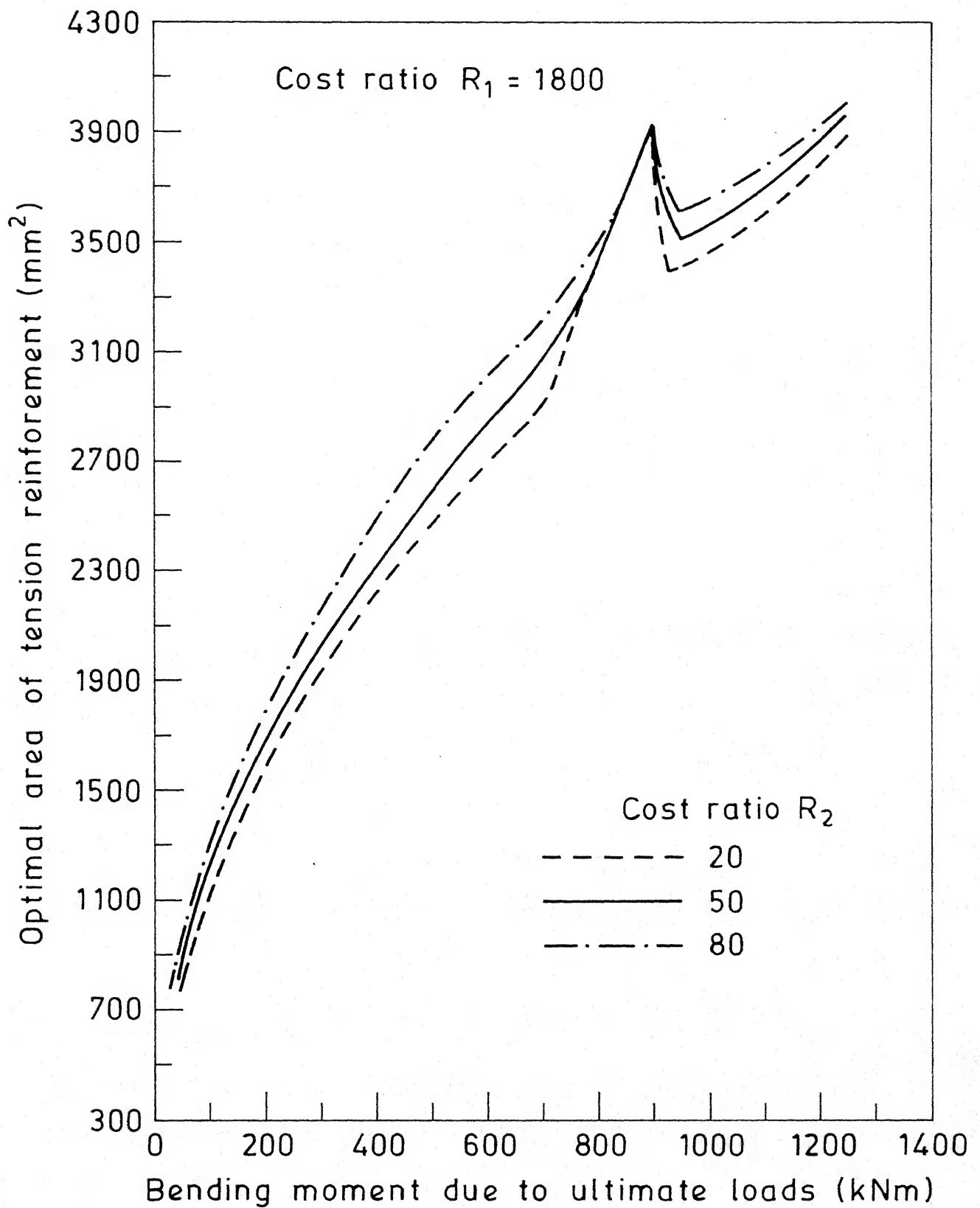
Fig. 2.10 - Disign chart for optimal area of tension reinforcement for beam.

Fig. 2.11 - Design chart for optimal area of tension reinforcement for beam.

constant at 750 mm even with an increase in the value of M,
till the cost of extra tension reinforcement required exceeds
the combined costs of extra concrete, extra formwork, and
side face reinforcement. This happens only for large values
of M and therefore for most of the T-beam floors, 750 mm may
be taken as an optimal upper bound to the depth of beam.

Large depths of beams associated with their optimal
design, results in a high value for the moment of inertia.
Therefore, it is observed that deflection is never critical
in optimal design of T-beams. The high value of moment of
inertia is also responsible for crack widths to be small at
the bottom face of the beam; cracks on the sides of the beam
are taken care of by the side face reinforcement when the
depth of beam exceeds 750 mm.

The constraints which are active in the optimal
design of a T-beam floor are listed below:

(1)   constraint g_2, regarding the moment of resistance at
        support B for the slab, when the loads are heavy and
        spans large;

(2)   constraint g_3, dealing with the moment of resistance
        of slab at C;

(3)   constraint g_4, controlling the deflection of slab;

(4)   constraint g_5, specifying a minimum percentage of
        reinforcement over support B; and

(5)   constraint g_8, regarding the moment of resistance
        of beam.

2.9   Method of Using the Design Charts

2.9.1   Procedure

The following procedure may be adopted to utilize the design charts developed and arrive at optimal values of the design variables for a given problem:

(1)   Span of the slab l_s, span of the beam l_b, magnitude of imposed load q_k, cost of finished concrete per m^3, cost of reinforcement per newton, and cost of formwork per m^2 are identified.

(2)   For the known values of l_s and q_k, optimal thickness of slab h_s (mm), and optimal areas of reinforcement a_{s1} and a_{s3} (mm^2/m) are determined from Figures 2.3 through 2.5. Optimal value of a_{s2} (mm^2/m) = 3 h_s.

(3)   Cost ratios R_1 and R_2 are evaluated.

(4)   With the known value of h_s and an assumed value of h_b, the maximum bending moment in the beam at ultimate loads, M, is computed in kNm.

(5)   Depending on the values of R_1 and R_2, appropriate design charts are selected and optimal values of A_{st} (mm^2) and h_b (mm) are determined. When necessary, interpolation may be used.

(6)   Width of rib may be chosen as 150 mm; or 160 mm if 32 mm diameter bars are to be used.

(7)   A detailed analysis is carried out to modify the design variables, if found necessary, and to fill in the other details.

2.9.2   Example

The method of using the design charts is illustrated by the following example.

Data : l_s = 3900 mm, l_b = 8780 mm, q_k = 3 kN/m^2, grade of concrete 25, cost of finished concrete per m^3 = 480 rupees, cost of reinforcement per newton = 0.4 rupee, cost of formwork per m^2 = 20 rupees.

From Figures 2.3 through 2.5, optimal values of h_s, a_{s1} and a_{s3}, for a slab of 3900 mm span subjected to an imposed load of 3 kN/m^2, are obtained as 117 mm, 600 mm^2/m and 220 mm^2/m respectively.

The slab thickness may be rounded off to 120 mm which leads to a value of a_{s2} = 360 mm^2/m. Since the slab thickness has been increased by 3 mm it is possible to reduce the values of a_{s1} and a_{s3}. By carrying out an elastic analysis of the slab, it is found that the required values of a_{s1} and a_{s3} are 530 mm^2/m (from deflection criterion) and 205 mm^2/m (from strength consideration) respectively.

The cost ratios are given by R_1 = 480/0.4 = 1200 and R_2 = 20/0.4 = 50.

Assuming a beam depth of 700 mm and a rib width of 160 mm, the ultimate load on the beam = 1.4 x [3900 x 120 + (700 - 120) x 160] x 24 x 10^{-6} + 1.6 x 3 x 10^{-3} x 3900 = 37.563 N/mm.

Maximum bending moment in the beam due to ultimate loads = 37.563 x $8780^2/8$ = 362 x 10^6 N mm = 362 kNm.

Corresponding to M = 362, R_1 = 1200 and R_2 = 50, optimal depth of beam = 585 mm (from Figure 2.7) and optimal area of tension reinforcement = 1940 mm^2 (from Figure 2.10). Choosing A_{st} = 1963 mm^2 (4 bars of 25 mm), required values of h_b and b_w are 575 mm and 150 mm respectively.

CHAPTER 3

OPTIMAL DESIGN OF SURFACE WATER TANKS WITHOUT ROOF

3.1 Design Considerations

3.1.1 General

Water tanks are constructed below, at or above the ground level, depending on the requirements of service. They may be built with or without a roof. The circular and rectangular shapes are more common than others. While cylindrical tanks are essentially subjected to hoop tension, rectangular tanks are subjected to both bending moment and tension. But, the economy of cylindrical tanks due to their superior structural form is sometimes offset by the expensive formwork required to build them. The structure considered in this chapter is a surface cylindrical water tank which is open at the top.

3.1.2 Materials

The recommended grades of concrete for reinforced concrete structures, according to CP 110, are 25 and 30. In view of the early thermal cracking problem associated with richer mixes, concrete of grade 25 is preferred in the current study. Traditionally mild steel reinforcement is used for water tanks because of the low allowable stresses permitted in their design by the working stress method.

But the superior bond properties of deformed bars, allowing
shorter laps and providing better crack distribution, will
compensate for their slight extra cost even if their higher
strength cannot be taken into account. The saving in area
of reinforcement will be substantial when the designs are
done according to limit state philosophy which permits the
use of higher stresses for strength calculations. Therefore,
deformed bars with a characteristic strength of 425 MPa have
been adopted. This value of characteristic strength has
been chosen for the sake of uniformity although a lower value
will not affect the design in any way.

3.1.3 Working stress method

Water tank is one structure which has, from the very
beginning, been designed from the aspect of serviceability,
that it should not crack and allow water to leak. The
following measures, in the design procedure, have been adopted
to ensure watertightness:

(1)   The thickness of the members is chosen such that the
       direct tensile stress as well as flexural tensile
       stress in concrete, computed on the equivalent
       uncracked section basis, do not exceed the permissible
       values.

(2)   The hoop reinforcement is designed to resist the
       complete hoop tension using a permissible stress of
       about 100 MPa.

But the superior bond properties of deformed bars, allowing shorter laps and providing better crack distribution, will compensate for their slight extra cost even if their higher strength cannot be taken into account. The saving in area of reinforcement will be substantial when the designs are done according to limit state philosophy which permits the use of higher stresses for strength calculations. Therefore, deformed bars with a characteristic strength of 425 MPa have been adopted. This value of characteristic strength has been chosen for the sake of uniformity although a lower value will not affect the design in any way.

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(1) The thickness of the members is chosen such that the direct tensile stress as well as flexural tensile stress in concrete, computed on the equivalent uncracked section basis, do not exceed the permissible values.

(2) The hoop reinforcement is designed to resist the complete hoop tension using a permissible stress of about 100 MPa.

(3)   The reinforcement required to resist bending moment
      is based on the cracked section, again with the same
      value of permissible stress.

      Such a design procedure is not only illogical but
also results in relatively thick concrete sections and subs-
tantial quantities of reinforcement due to the low working
stresses recommended for both the materials.

3.1.4   Limit state method

      The appropriate limit states for water retaining
structures are the serviceability limit state of cracking
and the ultimate limit state of strength. The check against
reaching the ultimate limit state is based on the limit
analysis, which is described later in Section 3.3. The limit
state of cracking is considered herein.

      Experience in the United Kingdom and elsewhere shows
that cracks of limited width do not necessarily allow the
liquid to leak, or if some percolation of liquid occurs on
first filling, the crack is sealed by autogeneous healing
(Anchor, 1977). It is therefore reasonable to approach the
design problem by postulating that cracks of limited width
can be allowed.

      The complex and semi-random phenomenon of cracking
has been tackled through experimental work. Beeby (1966,
1971, 1979), Broms (1964, 1965a, 1965b), Desai (1976a,
1976b, 1980) and Nawy (1970, 1971, 1972) are among the

investigators who have contributed significantly to the present state of art. Based essentially on the Dr. Beeby's work, BS 5337 has given the following equation for the prediction of surface flexural crack widths:

$$w_{cr} = \frac{4.5 \, a_{cr} \, \varepsilon_m}{1 + 2(\dfrac{a_{cr} - c_{min}}{h - d_n})} \, , \qquad ..(3.1)$$

where $\quad \varepsilon_m = \dfrac{\varepsilon_s y}{d - d_n} - \dfrac{0.7 \, b_t hy}{A_s (h - d_n) f_s} \times 10^{-3}$. $\qquad (3.2)$

The foregoing Eqs. (3.1) and (3.2) are of the same form as Eqs. (2.2) and (2.3). The difference is in the use of a coefficient 4.5 in Eq. (3.1) instead of 3.0 in Eq. (2.2) and a factor $0.7/f_s$ in Eq. (3.2) instead of $1.2/f_y$ in Eq. (2.3). Stress in the reinforcement at service loads is denoted by f_s. The difference between the two sets of equations is because of the probability of width of a crack exceeding the value given by Eq. (3.1) is about 5%, whereas in the case of Eq. (2.2) it is considerably higher. The greater margin of safety, against cracking, required for water retaining structures is reflected in this.

The permitted width of crack in water retaining structures depends on the class of exposure. Exposure class A is the most severe condition and applies to the region which is continuously exposed to saturated water vapour. The lower surface of a reservoir roof falls in this category. Exposure class B applies to regions which are always or

nearly always in contact with water. Members which do not come in contact with water, like the staging of an overhead tank, are designed for class C exposure. The maximum permissible surface crack widths for the three classes of exposure A, B and C are 0.1 mm, 0.2 mm and 0.3 mm respectively.

BS 5337 does not give any guidance for the calculation of crack widths in members subjected to direct tension. An equation, which can be used with the confidence required for the design of water retaining structures, is not yet available. Therefore, safety in respect of cracking due to direct tension is taken care of by the 'deemed to satisfy' provision, which restricts the permissible stress in deformed bars to 100 MPa for class A exposure and to 130 MPa for class B exposure.

3.2   Elastic Analysis

In order to find out the stresses and strains required for the computation of crack widths at service loads, elastic analysis is carried out. The Figure 3.1 shows the forces acting on a cylindrical shell element and the sign convention adopted. All the quantities are positive in the directions indicated. The notation adopted is as follows:

w = radial displacement of the shell,

p_x = water pressure,

M_x = bending moment in the vertical direction,
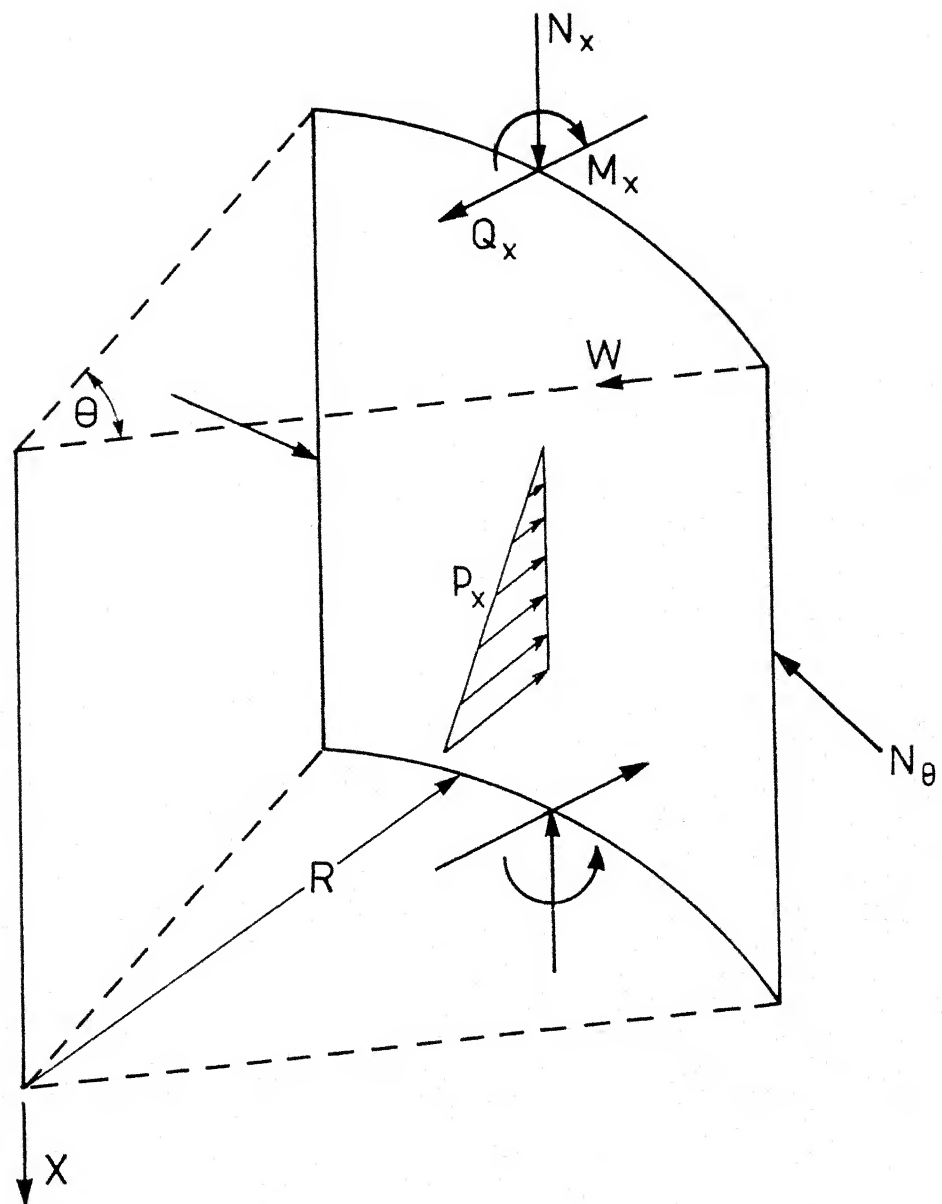
Q_x = shear force,

Fig. 3.1 - Forces acting on a cylindrical shell element.

N_x = longitudinal membrane force,

N_Θ = hoop force, all considered at any section x, and

R = radius of the shell.

Consideration of the equilibrium of the shell element results in the equation

$$\frac{d^2 M_x}{dx^2} = p_x + \frac{N_\Theta}{R} .$$ ..(3.3)

Both M_x and N_Θ can be expressed in terms of w as

$$M_x = - EI \frac{d^2 w}{dx^2} ,$$ ..(3.4)

and $$N_\Theta = \frac{Et}{R} w .$$ ..(3.5)

Thus, $$p_x = \gamma_w x = - \frac{wEt}{R^2} - EI \frac{d^4 w}{dx^4} ,$$ ..(3.6)

where E = modulus of elasticity of concrete,

I = second moment of area,

t = thickness of the wall, and

γ_w = specific weight of water.

Rearranging the Eq. (3.6), the differential equation of equilibrium is obtained in the form

$$\frac{d^4 w}{dx^4} + 4\mu^4 w = - \frac{\gamma_w x}{EI} ,$$ ..(3.7)

where $\mu^4 = t/4R^2 I = 3/R^2 t^2$.

The solution of Eq. (3.7) is given by

$$w = - \gamma_w x/4EI \; \mu^4 + e^{\mu x} (c_1 \sin \mu x + c_2 \cos \mu x)$$
$$+ e^{-\mu x} (c_3 \sin \mu x + c_4 \cos \mu x) , \qquad ..(3.8)$$

where c_1, c_2, c_3 and c_4 are the constants of integration. These are evaluated using the boundary conditions applicable to any particular problem considered.

3.2.1 Tank wall with a hinged base

For a tank wall which is free at the top and hinged at the bottom, the boundary conditions are given by

$$EI \left. \frac{d^2 w}{dx^2} \right|_{x = 0} = 0 , \qquad ..(3.9)$$

$$EI \left. \frac{d^3 w}{dx^3} \right|_{x = 0} = 0 , \qquad ..(3.10)$$

$$\left. w \right|_{x = 1} = 0 , \qquad ..(3.11)$$

$$\text{and} \quad EI \left. \frac{d^2 w}{dx^2} \right|_{x = 1} = 0 , \qquad ..(3.12)$$

where 1 is the length of the wall (depth of the tank).

The first and higher order derivatives of w are given by

$$\frac{dw}{dx} = -\frac{\gamma_w}{4EI\ \mu^4} + \mu e^{\mu x}\left[(c_1 - c_2)\sin \mu x + (c_1 + c_2)\cos \mu x\right]$$

$$-\mu e^{-\mu x}\left[(c_3 + c_4)\sin \mu x + (c_4 - c_3)\cos \mu x\right]\,, \qquad ..(3.13)$$

$$\frac{d^2 w}{dx^2} = 2\mu^2\left\{e^{\mu x}\left[(c_1 \cos \mu x - c_2 \sin \mu x)\right] - e^{-\mu x}\left[(c_3 \cos \mu x\right.\right.$$

$$\left.\left. - c_4 \sin \mu x)\right]\right\}, \qquad\qquad ..(3.14)$$

and

$$\frac{d^3 w}{dx^3} = 2\mu^3\ e^{\mu x}\left[(c_1 - c_2)\cos \mu x - (c_1 + c_2)\sin \mu x\right]$$

$$+ 2\mu^3\ e^{-\mu x}\left[(c_3 + c_4)\cos \mu x + (c_3 - c_4)\sin \mu x\right]. \quad ..(3.15)$$

Using Eqs. (3.14), (3.15) and (3.8) respectively, Eqs. (3.9), (3.10) and (3.11) are solved to yield the following relationships:

$$c_3 = c_1\,, \qquad\qquad ..(3.16)$$

$$c_4 = c_2 - 2c_1\,, \qquad\qquad ..(3.17)$$

and $\qquad c_2 = a + bc_1\,, \qquad\qquad ..(3.18)$

where $\qquad a = \dfrac{\gamma_w l}{4EI\ \mu^4(e^{\mu l} + e^{-\mu l})\cos \mu l}\,, \qquad ..(3.19)$

and $\qquad b = \dfrac{(2e^{-\mu l}\cos \mu l - e^{\mu l}\sin \mu l - e^{-\mu l}\sin \mu l)}{(e^{\mu l} + e^{-\mu l})\cos \mu l}.$

$$..(3.20)$$

The Eq. (3.12) is solved using Eqs. (3.14), (3.16), (3.17) and (3.18) to determine c_1 as

$$c_1 = \frac{a \sin \mu l (e^{\mu l} - e^{-\mu l})}{e^{\mu l}(\cos \mu l - b \sin \mu l) - e^{-\mu l}(\cos \mu l - b \sin \mu l + 2\sin \mu l)}$$

..(3.21)

The other constants of integration are easily evaluated using Eqs. (3.16), (3.18) and (3.17).

3.2.2   Tank wall with a semi-rigid base

When the wall of tank is built monolithically with the base, there will be certain amount of restraint against free rotation of the wall. It is difficult to predict the degree of restraint as it depends on the behaviour of the floor slab provided with movement joints at regular intervals and subjected to water pressure from the top and contact pressure from the bottom, the distribution of which is uncertain. It is generally recognized that the amount of actual restraint will correspond to something in between the fixed and the hinged conditions. If the analysis is done on the basis of a fixed base, the maximum hoop tension will be less and the maximum moment will be more. On the other hand, a hinged base condition leads to a larger value of hoop tension and a smaller value of maximum moment. A conservative approach to the problem that is sometimes followed is to design the vertical reinforcement to resist the moment on the fixed base condition and to design the horizontal reinforcement to resist hoop tension on the hinged base condition. The following analysis does not assume either

a fixed or a hinged base. On the other hand, the analysis is developed with a base which has a variable degree of fixity.

In the first stage, the tank wall is analysed assuming the base to be hinged as explained in the previous section. Using Eq. (3.13) the slope of the elastic curve at the base, θ, is computed as

$$\theta = \left.\frac{dw}{dx}\right|_{x=1} = \mu e^{\mu l} \left[(c_1 - c_2)\sin \mu l + (c_1 + c_2)\cos \mu l \right]$$

$$- \mu e^{-\mu l} \left[(c_3 + c_4)\sin \mu l + (c_4 - c_3)\cos \mu l \right]$$

$$- \frac{\gamma_w}{4EI\,\mu^4} \, . \qquad\qquad ..(3.22)$$

For a semi-rigid base, denoting the degree of fixity by β, slope at the base is given by $(1 - \beta)\theta$. In the second stage, the tank is analysed with the revised boundary conditions. Of the four boundary conditions, those given by Eqs. (3.9) through (3.11) remain unaltered. The fourth boundary condition is now given by

$$\left.\frac{dw}{dx}\right|_{x=1} = (1 - \beta)\theta \, . \qquad\qquad ..(3.23)$$

The degree of fixity β can take any value between 1 and 0, the extreme values corresponding to the fixed and hinged conditions. With this modified boundary condition, the constant of integration c_1 is now given by

$$c_1 = $$

$$\frac{(1-\beta)\theta + \dfrac{\gamma_W}{4EI \, \mu^4} + \mu a \sin \mu l (e^{\mu l}+e^{-\mu l}) - \mu a \cos \mu l (e^{\mu l}-e^{-\mu l})}{\mu(1-b) \sin \mu l (e^{\mu l}+e^{-\mu l}) + \mu \cos \mu l \left[(1+b)e^{\mu l} + (3-b)e^{-\mu l} \right]}$$

$$..(3.24)$$

The other constants of integration are determined again with the help of Eqs. (3.16), (3.18) and (3.17). With the known deflection configuration, the values of M_x and N_θ at any section can be obtained from Eqs. (3.4) and (3.5) using Eqs. (3.14) and (3.8). The maximum value of positive bending moment obviously occurs at $x = 1$. The location of maximum hoop tension and maximum negative moment can be determined by equating the slope of the elastic curve and the shear force to zero. The resulting transcendental equations have been solved numerically by the Newton-Raphson method.

3.3   Limit Analysis

3.3.1   Applicability of limit analysis

The general theory of limit analysis of rigid plastic structures has been rigorously formulated by Prager (1952). Methods of the theory of limit analysis can be applied to a broader class of structures like those composed of elastic-plastic materials or brittle-plastic materials. This would be possible, if it is first established experimentally that such structures deform considerably at practically

constant load intensity before collapse. It should also be possible to formulate the conditions of failure.

In the case of reinforced concrete shells, because the percentage of reinforcement is small, plastic behaviour is fairly well established. Experimental investigations, by Baker (1953), Ernest and Marlette (1954), Ovetchkin (1961), and Sawczuk (1961), strongly indicate that collapse mechanisms and limit loads do exist for reinforced concrete shells. In order to calculate the load carrying capacity of the shell, the following two conditions, corresponding to the yield condition and flow rule for rigid-plastic materials, need to be established:

(1)   A relationship between the strength or yield properties of a material in question and the critical combinations of the stress resultants which the material can just withstand.

(2)   The rule of instantaneous deformation for those zones of structure where the stress resultants satisfy such a relationship.

The failure criterion adopted for the current study is based on the work of Sawczuk and Olszak (1961) and is briefly described in the next section.

3.3.2   Criterion of failure

For an axisymmetric shell, there are in general four stress resultants; the bending moment M_x, the shear force

Q_x and membrane forces N_x and N_Θ. When Q_x is neglected in the condition of failure, the failure criterion is represented by two parabolic cylinders bounded by two parallel planes. If the longitudinal force N_x is small in comparison with M_x and N_Θ, which is true for the class of problems considered in the current study, the intersection of the cylinders with the plane $N_x = 0$ gives the simplified failure criterion in the $M_x - N_\Theta$ plane. The Figure 3.2 shows the simplified failure criterion and the rule of instantaneous deformation. In Figure 3.2,

m_u = ultimate moment of resistance per unit width, to
      resist positive moment,

m_u' = ultimate moment of resistance per unit width, to
      resist negative moment,

n_c = hoop compression capacity per unit width,

n_t = hoop tension capacity per unit width, all pertaining
      to any section of the shell,

κ_x = generalized strain associated with M_x, and

λ_Θ = generalized strain associated with N_Θ.

The moment and membrane force capacities can vary from section to section, but the profile of the failure criterion retains essentially the same shape.

3.3.3   Collapse load

     In the limit analysis of structures, it is a common practice to get a lower bound on the load carrying capacity
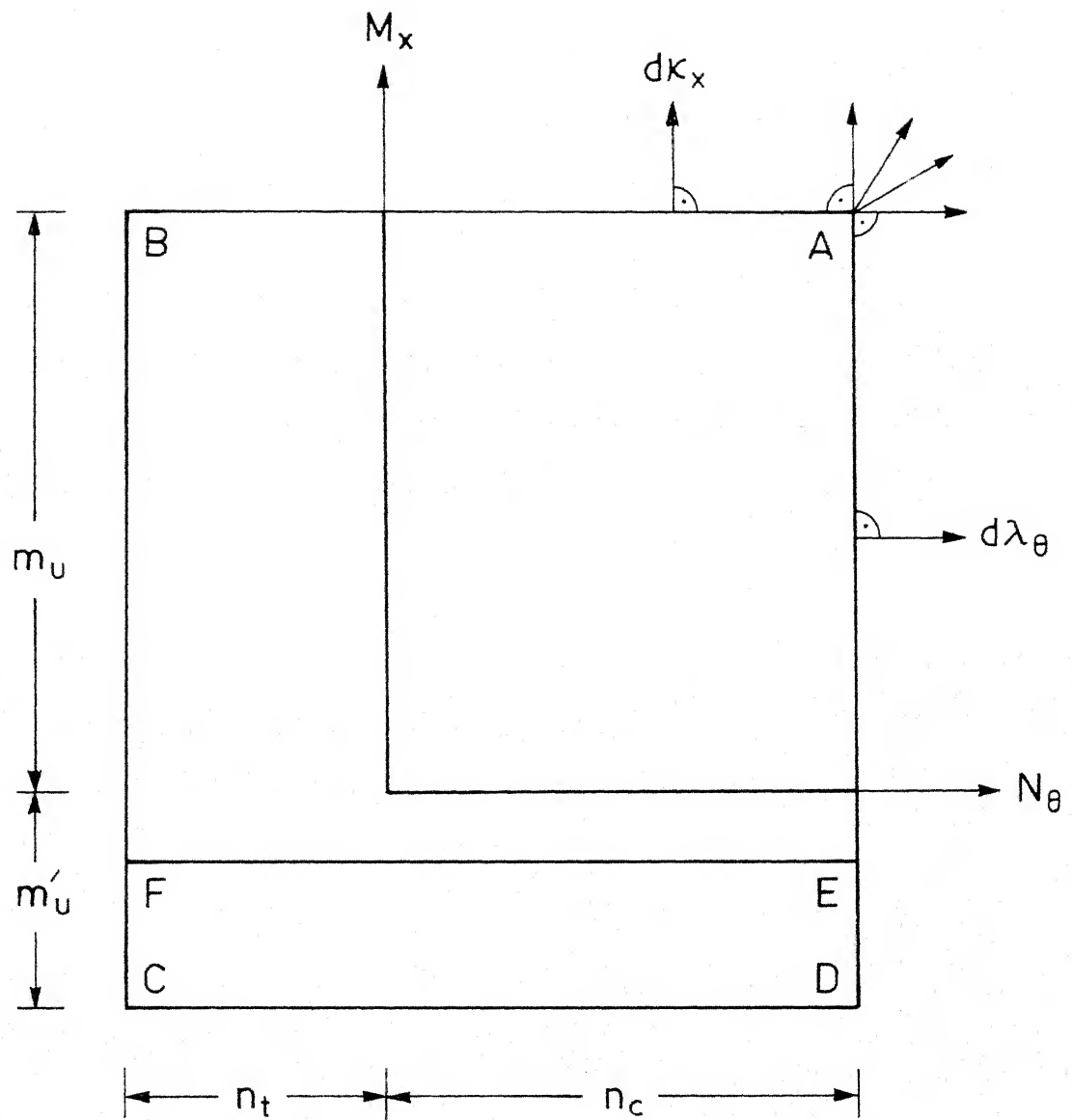
Fig. 3.2 - Simplified failure criterion.

from a statically admissible stress field or to obtain an
upper bound found a kinematically admissible mechanism.
There are very few cases in the limit analysis of shells for
which the exact value of the limit load has been found. It
would be safer to adopt the lower bound approach in the
absence of an exact solution. In this study, the solution is
commenced from the equilibrium equation and a kinematically
admissible mechanism for the stress field satisfying the
equation of equilibrium is then found. Hence the solution
is complete and gives the exact limit load.

The equation of equilibrium for an axisymmetric
shell is given by Eq. (3.3). Any distribution of M_x and N_θ,
which satisfies this equation and for which the values of
stress resultants do not exceed the corresponding capacities
at any section, is statically admissible. In order to limit
the crack widths at service loads, more reinforcement must be
placed at locations where forces are larger as obtained from
the elastic analysis. Since hoop tensions are larger in the
central portion of the tank under consideration, the distri-
bution of hoop tension capacity chosen, shown in Figure 3.3,
reflects this. The common practice of curtailing the
reinforcement where forces are smaller leads to the stepped
distribution. The relative values of n_{t1}, n_{t2}, n_{t3} and l_1
and l_2 depend on the shell parameters.

The generalized strain rates $\dot{\kappa}_x$ and $\dot{\lambda}_\theta$ can be
expressed in terms of w in the form

$$\dot{\kappa}_x = -\frac{d^2\dot{w}}{dx^2} , \qquad \qquad ..(3.25)$$

and $\qquad \dot{\lambda}_\theta = \frac{\dot{w}}{R} . \qquad \qquad ..(3.26)$

From Figure 3.2 it can be seen that regimes AB and CD have to be rejected because along AB and CD, $\dot{w}$ is equal to zero and $\frac{d^2\dot{w}}{dx^2}$ is not equal to zero, which is impossible. Along BC and AD, $\dot{\kappa}_x$ is equal to zero which leads to possible collapse mechanisms of the form $\dot{w} = c_1 x + c_2$. Therefore the collapse mode is always conical in form. The exact shape of the collapse mechanism depends on the shell parameters, and moment and hoop tension capacity distributions provided. Based on these, shells may be classified as short, medium and long.

3.3.4   Collapse mechanism 1

For a short shell, the bending moment diagram at collapse and the resulting shape of the mechanism are shown in Figure 3.3, along with the assumed hoop tension capacity distribution. The value of N_θ jumps from n_{t1} to n_{t2} at the first region boundary $x = l_1$ and from n_{t2} to n_{t3} at the second region boundary $x = l_2$, which is permissible; but the bending moment and shear force have to be continuous.

Due to the hydrostatic loading, the value of p_x will be equal to $p_c x/l$, where p_c is the collapse pressure. Integration of the equilibrium Eq. (3.3), for the region 1 $(N_\theta = - n_{t1})$, yields
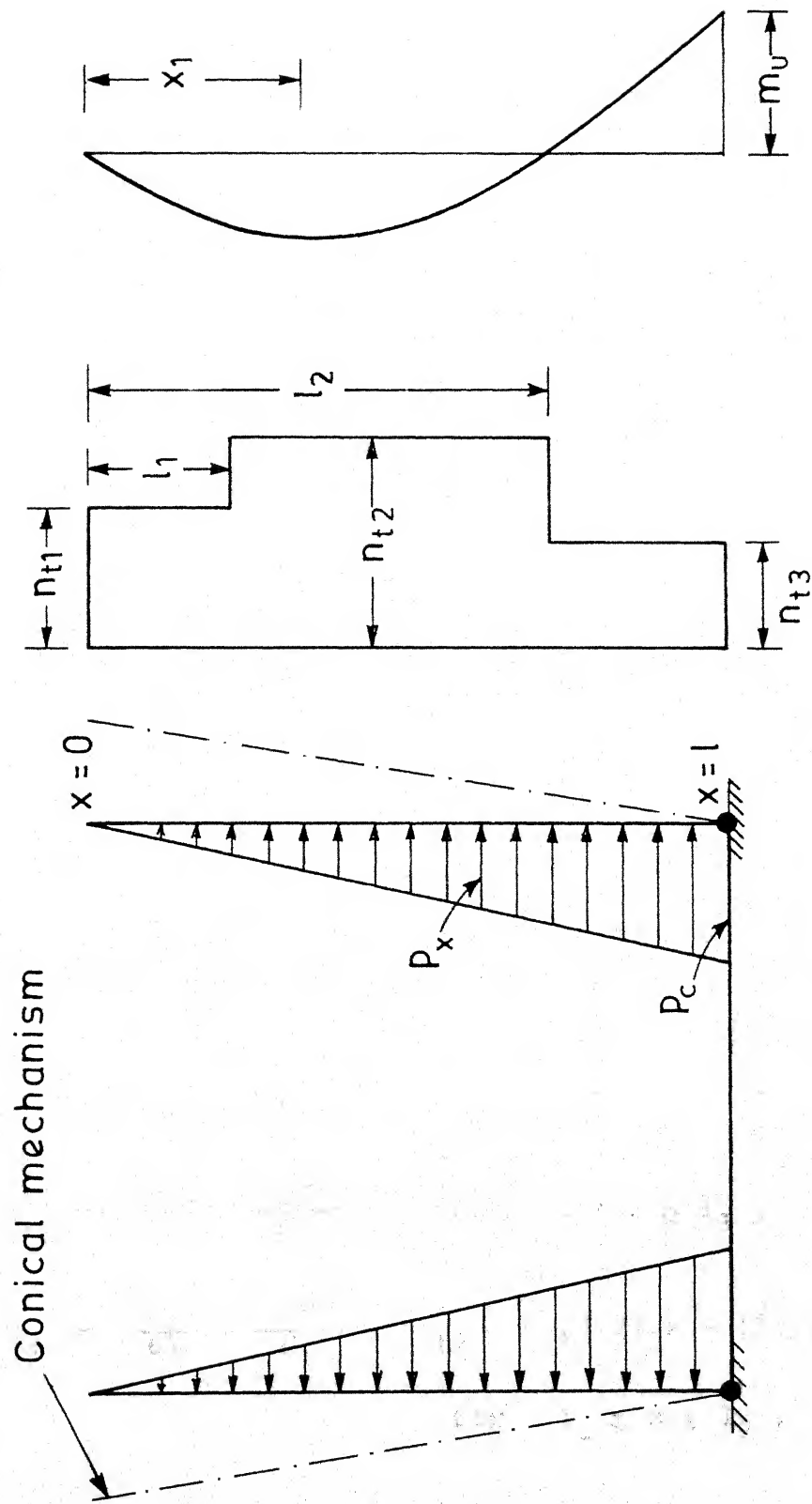
Fig. 3.3 - Details of collapse mechanism 1.

$$\frac{dM_x}{dx} + \frac{n_{t1}x}{R} - \frac{p_c x^2}{2l} + c_1 = 0 , \qquad ..(3.27)$$

$$\text{and} \qquad M_x + \frac{n_{t1}x^2}{2R} - \frac{p_c x^3}{6l} + c_1 x + c_2 = 0 . \qquad ..(3.28)$$

Similarly for region 2 ($N_\theta = - n_{t2}$)

$$\frac{dM_x}{dx} + \frac{n_{t2}x}{R} - \frac{p_c x^2}{2l} + c_3 = 0 , \qquad ..(3.29)$$

$$M_x + \frac{n_{t2}x^2}{2R} - \frac{p_c x^3}{6l} + c_3 x + c_4 = 0 , \qquad ..(3.30)$$

and for region 3 ($N_\theta = - n_{t3}$)

$$\frac{dM_x}{dx} + \frac{n_{t3}x}{R} - \frac{p_c x^2}{2l} + c_5 = 0 , \qquad ..(3.31)$$

$$\text{and} \qquad M_x + \frac{n_{t3}x^2}{2R} - \frac{p_c x^3}{6l} + c_5 x + c_6 = 0 . \qquad ..(3.32)$$

The constants of integration c_1 through c_6 are determined from the conditions that M_x and Q_x are equal to zero at $x = 0$ and are continuous at $x = l_1$ and $x = l_2$. After simplification, the following equations are obtained for M_x in the three regions:

$$M_x = \frac{p_c x^3}{6l} - \frac{n_{t1}x^2}{2R} \qquad \text{for} \quad 0 \leqslant x \leqslant l_1 , \qquad ..(3.33)$$

$$M_x = \frac{p_c x^3}{6l} - \frac{n_{t2}x^2}{2R} + (n_{t2} - n_{t1})(2l_1 x - l_1^2)/2R$$
$$\text{for} \quad l_1 \leqslant x \leqslant l_2 , \qquad ..(3.34)$$

3.3.5   Collapse mechanism 2

For total collapse of the shell, when the first mechanism is not possible, the failure will be as shown in Figure 3.4. The top portion of the tank will be in a state of hoop compression and the value of N_θ jumps from n_c to $-n_{t1}$ at the first region boundary $x = x_o$ as shown by the line EF in Figure 3.2. For this mechanism, there are four different regions. The hoop membrane forces in these four regions are as indicated below:

region 1 : $0 \leqslant x \leqslant x_o$ $N_\theta = n_c$,

region 2 : $x_o \leqslant x \leqslant l_1$ $N_\theta = - n_{t1}$,

region 3 : $l_1 \leqslant x \leqslant l_2$ $N_\theta = - n_{t2}$, and

region 4 : $l_2 \leqslant x \leqslant l$ $N_\theta = - n_{t3}$.

Integration of the equilibrium Eq. (3.3) is carried out separately for each of the four regions resulting in 8 constants of integration. These are evaluated from the conditions that M_x and Q_x are equal to zero at $x = 0$ and are continuous at the three region boundaries $x = x_o$, $x = l_1$ and $x = l_2$. The simplified equations for shear force and bending moment in the four regions are the following:

region 1 : $\dfrac{dM_x}{dx} = \dfrac{n_c x}{R} + \dfrac{p_c x^2}{2l}$,   ..(3.38)

$M_x = \dfrac{n_c x^2}{2R} + \dfrac{p_c x^3}{6l}$,   ..(3.39)

Fig. 3.4 - Details of collapse mechanism 2.

region 2 : $\dfrac{dM_x}{dx}$ $=$ $\dfrac{1}{R}\,[\dfrac{p_cRx^2}{2l} - n_{t1}x + (n_{t1} + n_c)x_o]$,   ..(3.40)

M_x $=$ $\dfrac{1}{2R}\,[\dfrac{p_cRx^3}{3l} - n_{t1}x^2 + 2(n_c + n_{t1})xx_o$

$- (n_{t1} + n_c)x_o^2]$,   ..(3.41)

region 3 : $\dfrac{dM_x}{dx}$ $=$ $\dfrac{1}{R}\,[\dfrac{p_cRx^2}{2l} - n_{t2}x + (n_{t2} - n_{t1})l_1$

$+ (n_{t1} + n_c)x_o]$,   ..(3.42)

M_x $=$ $\dfrac{1}{2R}\,[\dfrac{p_cRx^3}{3l} - n_{t2}x^2 + 2(n_{t2} - n_{t1})l_1x$

$+ 2(n_{t1} + n_c)x_ox - (n_{t2} - n_{t1})l_1^2$

$- (n_{t1} + n_c)x_o^2]$,   ..(3.43)

region 4 : $\dfrac{dM_x}{dx}$ $=$ $\dfrac{1}{R}\,[\dfrac{p_cRx^2}{2l} - n_{t3}x + (n_{t3} - n_{t2})l_2$

$+ (n_{t2} - n_{t1})l_1 + (n_{t1} + n_c)x_o]$,   ..(3.44)

and M_x $=$ $\dfrac{1}{2R}\,[\dfrac{p_cRx^3}{3l} - n_{t3}x^2 + 2(n_{t3} - n_{t2})l_2x$

$+ 2(n_{t2} - n_{t1})l_1x + 2(n_{t1} + n_c)x_ox$

$- (n_{t3} - n_{t2})l_2^2 - (n_{t2} - n_{t1})l_1^2$

$- (n_{t1} + n_c)x_o^2]$.   ..(3.45)

In order to solve for the three unknowns p_c, x_o and x_1, the following conditions are used:

at $x = 1$ (region 4) $M_x = m_u$,   ..(3.46)

at $x = x_1$ (region 3) $M_x = - m_u'$,   ..(3.47)

and   at $x = x_1$ (region 3) $\dfrac{dM_x}{dx} = 0$.   ..(3.48)

The foregoing three conditions, when used along with Eqs. (3.45), (3.43) and (3.42) respectively, result in three non-linear simultaneous equations. These have been solved numerically using the Newton-Raphson method.

For the mechanism to be valid, the value of moment at $x = x_2$ should not exceed the value of m_u at that section. The value of x_2 can be found by equating Eq. (3.40) to zero.

3.3.6   Collapse mechanism 3

When failure of the shell by collapse mechanisms 1 or 2 is ruled out, total collapse of the shell is not possible and the failure is by partial collapse of the bottom portion of the shell. Details of this mode of collapse are shown in Figure 3.5. Positive hinge circles form at $x = 0$ and $x = x_2$ and a negative hinge circle at $x = x_1$. Beyond $x = x_2$, the bending moment diagram can be continued in a statically admissible manner. Since the origin for this mechanism is chosen at the bottom, the equation of equilibrium will now be
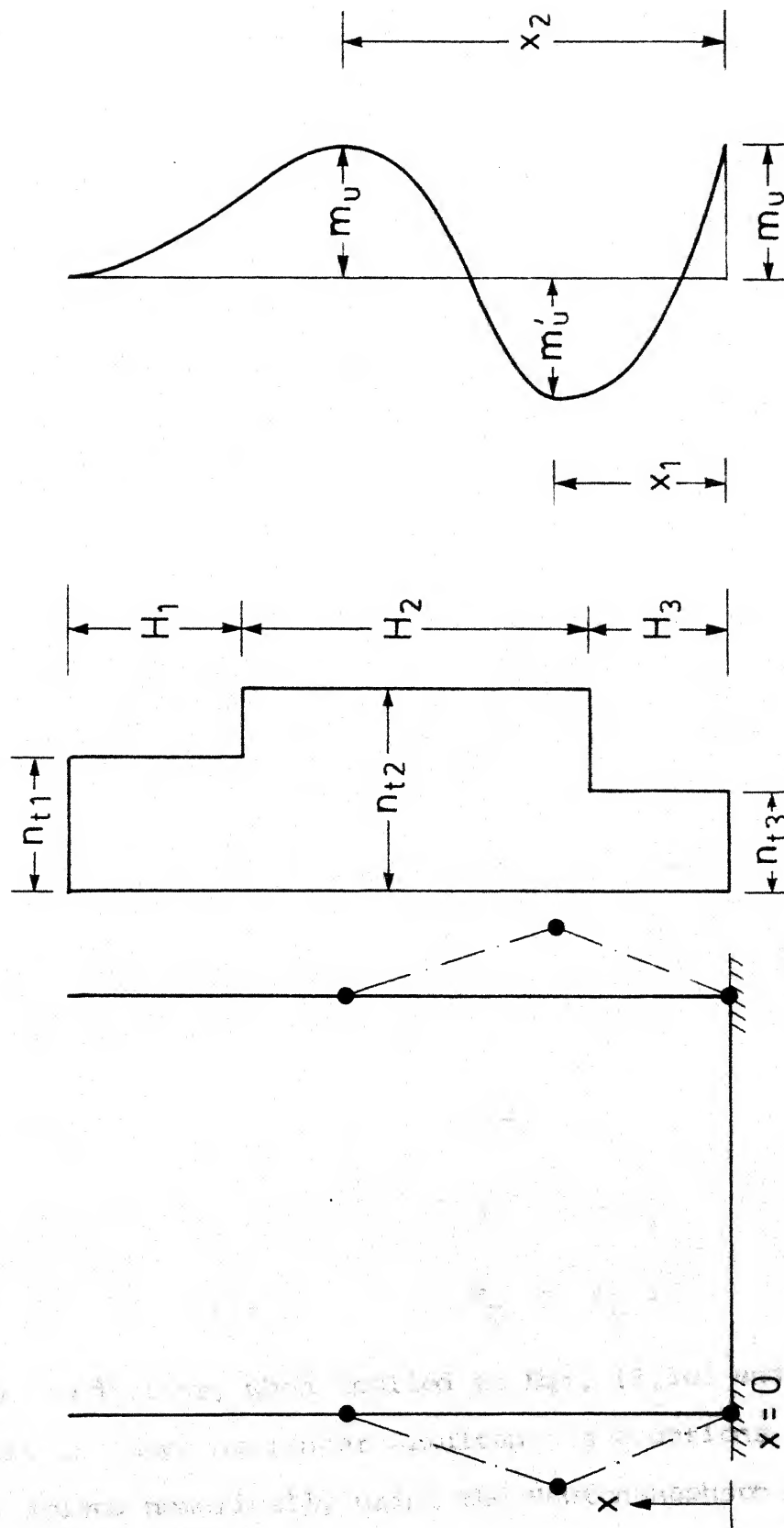
Fig. 3.5 - Details of collapse mechanism 3.

$$\frac{d^2 M_x}{dx^2} - \frac{N_\theta}{R} - p_c(1 - \frac{x}{l}) = 0 . \qquad ..(3.49)$$

Integration of this equation, in the bottom two regions, results in 4 constants of integration. In addition to the moment and shear continuity conditions at the first region boundary, the conditions that $M_x = m_u$ at $x = 0$ and $\frac{dM_x}{dx} = 0$ at $x = x_2$ are used to evaluate these constants of integration. After simplification, the shear and moment in the middle region are given by the following equations:

$$\frac{dM_x}{dx} = \frac{p_c}{2l} (2lx - x^2 - 2lx_2 + x_2^2) - \frac{n_{t2}}{R} (x - x_2) ,$$

$$..(3.50)$$

and
$$M_x = \frac{p_c}{6l} (3lx^2 - x^3 - 6lx_2 x + 3x_2^2 x)$$

$$- \frac{n_{t2}}{2R} (x^2 - 2x_2 x + l_3^2) + m_u + \frac{n_{t3} l_3^2}{2R} . \qquad ..(3.51)$$

In order to solve for the three unknowns p_c, x_1 and x_2, the following conditions are used:

at $x = x_1$, $\qquad \frac{dM_x}{dx} = 0$, $\qquad ..(3.52)$

at $x = x_1$, $\qquad M_x = - m_u'$, $\qquad ..(3.53)$

and at $x = x_2$, $\qquad M_x = m_u$. $\qquad ..(3.54)$

These conditions, when applied to Eqs. (3.50) and (3.51), result in three nonlinear simultaneous equations which have been solved numerically using the Newton-Raphson method.

It has been assumed that both x_1 and x_2 would be within the middle region ($N_\theta = - n_{t2}$). After solving for the values of x_1 and x_2, if it is found that either x_1 lies in the bottom region or x_2 in the top region, then the equations for shear and moment have to be written for these regions and the simultaneous equations obtained using the appropriate conditions.

3.4   Optimization

3.4.1   Objective function

The cost of finished concrete, reinforcement and formwork required for the tank wall as well as the floor is chosen as the objective function. The floor of surface water tanks generally consists of a floor slab provided with complete movement joints at regular intervals and a layer of blinding concrete with a separating layer of thick polythene or thick bituminous material between the floor slab and the blinding layer. Since the forces induced in such a floor slab are not significant, it does not warrant any design. It is assumed that the tank floor consists of a 150 mm thick floor slab with 0.2% reinforcement in both directions and a 75 mm layer of blinding concrete. Since the blinding concrete will be of a lower grade, its equivalent thickness is taken as 60 mm. In order to include the effect of thickened edges and cost of joints, cost of the floor is increased by 10%.

The expression for the objective function F can be written as,

$$F = R_1 (1.1 \pi R^2 \times 210 + 2 \pi R t l) + 2 \pi R \gamma_s (1.1 \times 0.002 R$$

$$\times 150 + a_{h1} H_1 + a_{h2} H_2 + a_{h3} H_3 + a_{v1} l_1 + a_{v2} l_2)$$

$$+ 4 \pi R l R_2 , \qquad\qquad ..(3.55)$$

where a_{hi} = area of hoop reinforcement, per unit width,
   provided over a height H_i,

and a_{vi} = area of vertical reinforcement, per unit width,
   provided over a length l_i.

3.4.2 Design variables

The design variables chosen are the radius of the tank wall, R, thickness of the wall, t, and the reinforcement areas a_{h2} and a_{v2} which denote the maximum values of the hoop and vertical reinforcements respectively. For fully or partially restrained walls, the value of maximum positive bending moment occurs at the bottom; however, this moment reduces at an exponential rate and hence the value of vertical reinforcement needs to be a_{v2} for a short length only. For the remaining length, the minimum reinforcement of 0.3% has been found to be more than adequate. The hoop tension will be maximum in the middle portion and less in the end regions. The hoop reinforcement to resist it can, theoretically, be varied continuously to arrive at the

least reinforcement volume. But, practical considerations prohibit such an approach and the hoop reinforcement is reduced stepwise in the top and bottom portions. The normal practice of curtailing the reinforcement by 50% is followed here also with the result that the ratio of both a_{h1} and a_{h3} to a_{h2} is 0.5. Other ratios were tried, but they did not lead to any better result. Therefore, heights at which the hoop tension is half of the maximum value are located and hoop reinforcement is curtailed by 50% beyond these heights.

3.4.3   The constraints

The first and second constraints deal with surface flexural crack widths, w_{cr1} and w_{cr2}, due to maximum positive and negative moments in the tank wall. These are computed using Eq. (3.1). Since the tank under consideration is open at the top, assuming class B exposure, the maximum permitted width of crack, w_{crp}, is taken as 0.2 mm. The first and second constraints are written in the form

$$\frac{w_{cr1}}{w_{crp}} - 1 \leq 0 , \qquad ..(3.56)$$

and
$$\frac{w_{cr2}}{w_{crp}} - 1 \leq 0 . \qquad ..(3.57)$$

Cracks due to hoop tension are controlled by using the 'deemed to satisfy' condition. For class B exposure, the permissible tensile stress, f_{max}, is 130 MPa for deformed bars. Therefore, the third constraint may be stated as

$$\frac{(N_\theta)_{max}}{a_{h2} f_{max}} - 1 \leqslant 0 .$$
..(3.58)

The limit analysis, carried out to assess the strength of the tank to resist the water pressure, furnishes the value of collapse pressure p_c. Using a partial safety factor 1.6 against collapse, the fourth constraint may be stated as

$$\frac{1.6 \, \gamma_w l}{p_c} - 1 \leqslant 0 .$$
..(3.59)

Practical considerations dictate a minimum thickness, t_{min}, for the tank wall. There is generally no advantage in adopting a thickness less than 150 mm - 125 mm as the precision required in fixing the reinforcement, placing and compacting concrete would involve extra cost. An error in the displacement of the reinforcement, provided to resist bending moment, will be more serious in thinner walls. A minimum thickness of 125 mm is adopted in the current study. Therefore, the fifth constraint can be written as

$$\frac{t_{min}}{t} - 1 \leqslant 0 .$$
..(3.60)

To take care of the temperature variations and shrinkage effects, the area of reinforcement in each of the two directions should not be less than a_{min}, which is equal to 0.3% as per BS 5337. Thus, constraints (6) through (8) can be expressed in the form

$$\frac{a_{min}}{a_{v1}} - 1 \leq 0, \qquad\qquad ..(3.61)$$

$$\frac{a_{min}}{a_{v2}} - 1 \leq 0, \qquad\qquad ..(3.62)$$

and $\qquad\qquad \frac{a_{min}}{a_{h1}} - 1 \leq 0. \qquad\qquad ..(3.63)$

Only for those design variables for which lower bounds have not been specified, non-negativity constraints must be stated. One such variable is the radius R. Although the height of the tank, H, is not explicitly taken as a design variable because it is obtained from the capacity of the tank and radius R, non-negativity constraint must be stated for heights H_1, H_2 and H_3. This is obvious from Eq. (3.55) for the objective function, as a negative value for any H_i will lead to a lesser value of the objective function. Thus, constraints (9) through (12) are written in the form

$$- \frac{R}{R_n} \leq 0, \qquad\qquad ..(3.64)$$

and $\qquad\qquad - \frac{H_i}{H} \leq 0, \quad$ for $(i = 1, 2, 3), \qquad ..(3.65)$

where R_n is a suitable normalizing radius.

3.5 Arrangement of Reinforcement

The placing of vertical reinforcement follows from the bending moment diagram at service loads. The vertical reinforcement to resist positive bending moment

should be placed as near to the water face as possible and to resist negative moment as away from the water face as possible. The cover provided is 40 mm which is the minimum as per BS 5337. A larger value of cover is not desirable as it might lead to wider cracks. In order to make provision for accidental reversal of bending moment, 80% of reinforcement is placed near the appropriate face and 20% near the opposite face. The computation of stresses is based on this arrangement. The hoop reinforcement is placed on the inner side of vertical reinforcement, 50% on each side, in a staggered fashion. In the limit analysis, depending on the position of the hinge circle and the areas of reinforcement provided there, values of m_u, m_u', and n_t are computed.

3.6   Example

The elastic analysis and limit analysis developed in Sections 3.2 and 3.3 are applied for the optimal limit state design of a water tank of capacity 750 m^3 and provided with a free board of 200 mm. The results are compared with an indirect working stress design for the same data, in order to bring out the difference between the two methods and also to get an idea of the savings obtained because of the application of limit state philosophy and optimization. The degree of fixity β has been assumed as 0.4. The optimal design has been obtained for $\beta = 0.4$ as well as for $\beta = 1$. Class of exposure is taken as B.

A number of trials were required in order to arrive at an efficient indirect design so that the materials are stressed to their full permissible value. The permissible stresses for concrete of grade 25 and deformed bars, as per BS 5337, are shown along with the computed values.

The following results were obtained from the elastic analysis carried out with R = 6.5 m, t = 185 mm, and β = 0.4:

maximum hoop tension = 282 kN/m,

maximum positive moment = 7.056 kNm/m,

maximum negative moment = 5.007 kNm/m,

heights H_1 and H_3 = 2.19 m and 0.51 m.

Areas of reinforcement provided are a_{v1} = a_{v2} = 555 mm^2/m (minimum value), a_{h2} = 2175 mm^2/m and a_{h1} = a_{h3} = 1088 mm^2/m. The resulting maximum stresses are:

direct tensile stress in concrete = 1.309 MPa < 1.31 MPa,

flexural tensile stress in concrete = 1.18 MPa < 1.84 MPa,

direct tensile stress in steel = 129.66 MPa < 130 MPa, and

flexural tensile stress in steel = 124.5 MPa < 130 MPa.

For the optimal design, the values of design variables depend on cost ratios R_1 and R_2. The Table 3.1 gives these values of the design variables and also a comparison of the values of the objective function for the indirect design with β = 0.4, optimal design with β = 0.4, named as OPD1, and optimal design with β = 1, named as OPD2. Since the design variables for the indirect design do not vary,

Table 3.1 Values of design variables and objective function

Cost ratio R_1	Cost ratio R_2	Design nomenclature	Radius R (m)	Thickness t (mm)	Reinforcement a_{v2} (mm²/m)	Reinforcement a_{h2} (mm²/m)	Objective function
600	100	OPD1	8.89	125	375	1215	116758 (145076)
		OPD2	8.98	125	525	1005	115064
1200	40	OPD1	7.44	125	375	1795	136286 (161262)
		OPD2	7.54	125	860	1595	135783
1200	100	OPD1	7.86	125	375	1605	161371 (190740)
		OPD2	8.00	125	735	1390	160426
1800	100	OPD1	7.32	125	375	1850	202138 (236403)
		OPD2	7.45	125	885	1635	201783

only the value of the objective function is given within brackets along with the value of the objective function for OPD1.

A number of observations can be made from the results of this example. For optimal designs, increase in the degree of fixity, cost ratios remaining same, results in a slightly larger radius, higher value of a_{v2} and lower value of a_{h2}. This is because of the increase in the value of positive moment. An increase in the value of R_2, other factors remaining same, would also result in a larger radius, so that the surface area would decrease. The thickness has assumed the minimum value in all cases.

The percentage increase in cost of indirect design with respect to optimal design varies from about 25% for the cost ratio combination (600, 100) to about 17% for the cost ratio combination (1800, 100). In view of the limiting tensile stresses in concrete in the working stress method, the wall needs to be thicker. This would not result in much saving of reinforcement as a minimum of 0.3% of concrete area has to be provided. The degree of fixity was intentionally chosen as 0.4. A larger value would have resulted in a higher moment and a smaller value would have resulted in a higher hoop tension, both demanding a thicker wall. Consequently the cost would have increased.

Thus considerable economy would result when advantage is taken of the provisions of limit state philosophy and benefits of optimization.

3.7   Parametric Study

Optimal limit state design of surface water tanks open at the top has been carried out for the following cases:

(1)   Capacity in m^3 :   250, 500, 750, 1000, 1250 and 1500.

(2)   Degree of fixity :   1.0, 0.8, 0.6, 0.4, 0.2 and 0.0.

(3)   Cost ratio R_1 :   600, 1200 and 1800.

(4)   Cost ratio R_2 :   40, 100 and 160.

A free board of 200 mm has been provided in all cases.

3.8   Results and Discussion

The results of the parametric study have been presented essentially in the form of design charts. It has been observed that for a given capacity, cost ratios being same, there is not much difference in the values of optimal radius for different degrees of fixity; the optimal radius is maximum when the base is fully fixed and gradually decreases with the decrease in the degree of fixity till a minimum is reached for a value of β around 0.4. Beyond this value, there is a tendency for slight increase in the optimal radius. This observation is true for all capacities and all cost ratios, though the value of β at which the

optimal radius reaches the minimum varies slightly. The Table 3.2 shows the effect of degree of fixity on the optimal radius for the cost ratio combination (1200, 100). It is readily seen that there is not much of a difference between the maximum radius and minimum radius for a given capacity; the maximum difference is about 2%, for smaller capacities it is less than this value. The design charts for optimal radius, given in Figures 3.6 through 3.8 for different cost ratio combinations, have been made using the minimum value optimal radius for any given capacity. When the degree of fixity is known to be more than 0.4, a slightly larger radius can be chosen.

From these design charts it can be observed that, for a given capacity, the optimal radius decreases with an increase in the value of R_1 and for the same value of R_1, it increases with an increase in the value of R_2. The cost of tank consists of the cost of tank wall and cost of floor. An increase in the radius, for a given capacity, results in a smaller surface area as well as reduced moments and hoop tensions. Cost of concrete, reinforcement and formwork for the tank wall, which are all proportional to the surface area and depend on the values of moments and hoop tensions, consequently decrease with an increase in the radius. On the other hand, cost of concrete and reinforcement required for the floor, which depend on the area of the floor, increase with the radius. Because of these conflicting
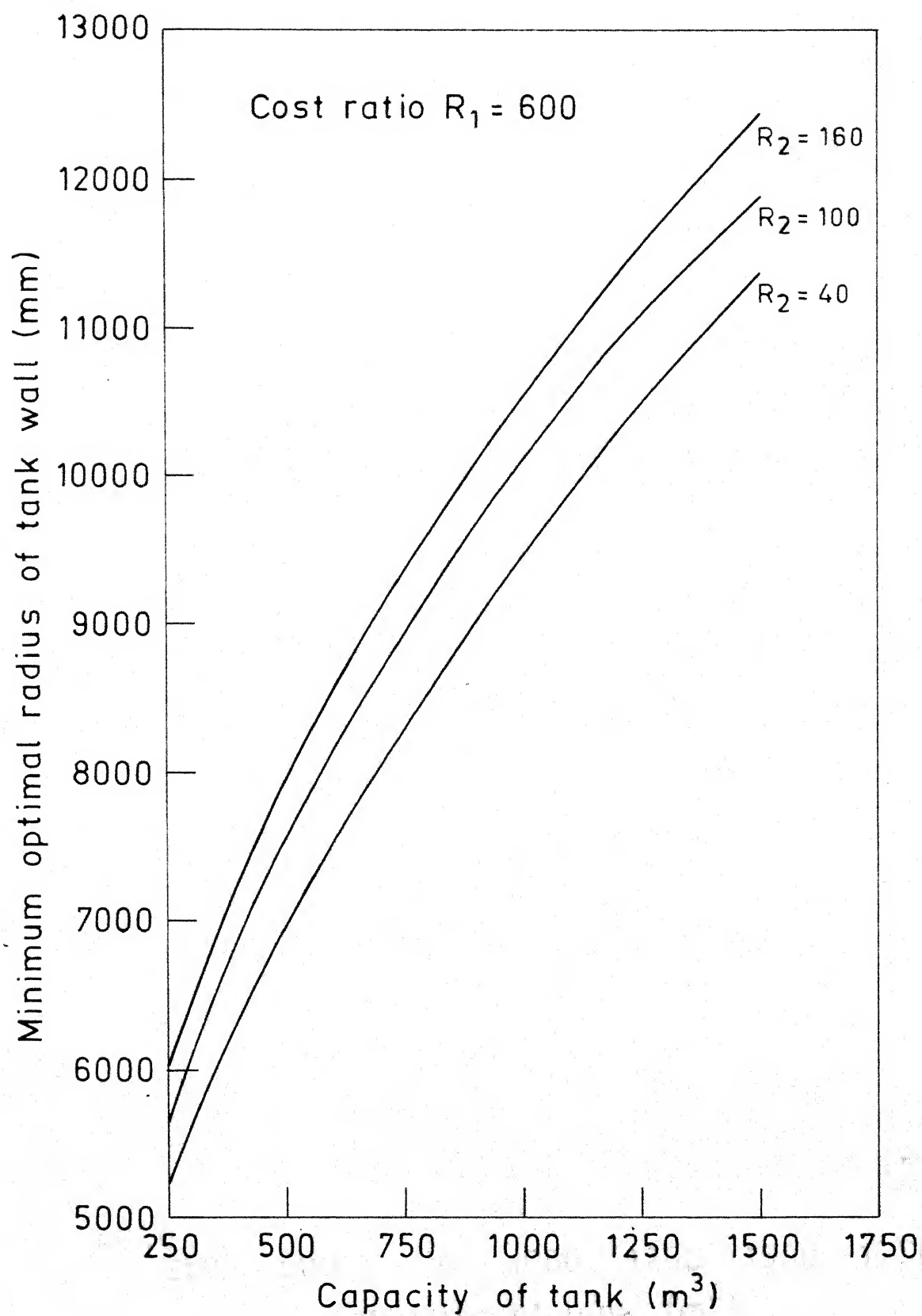
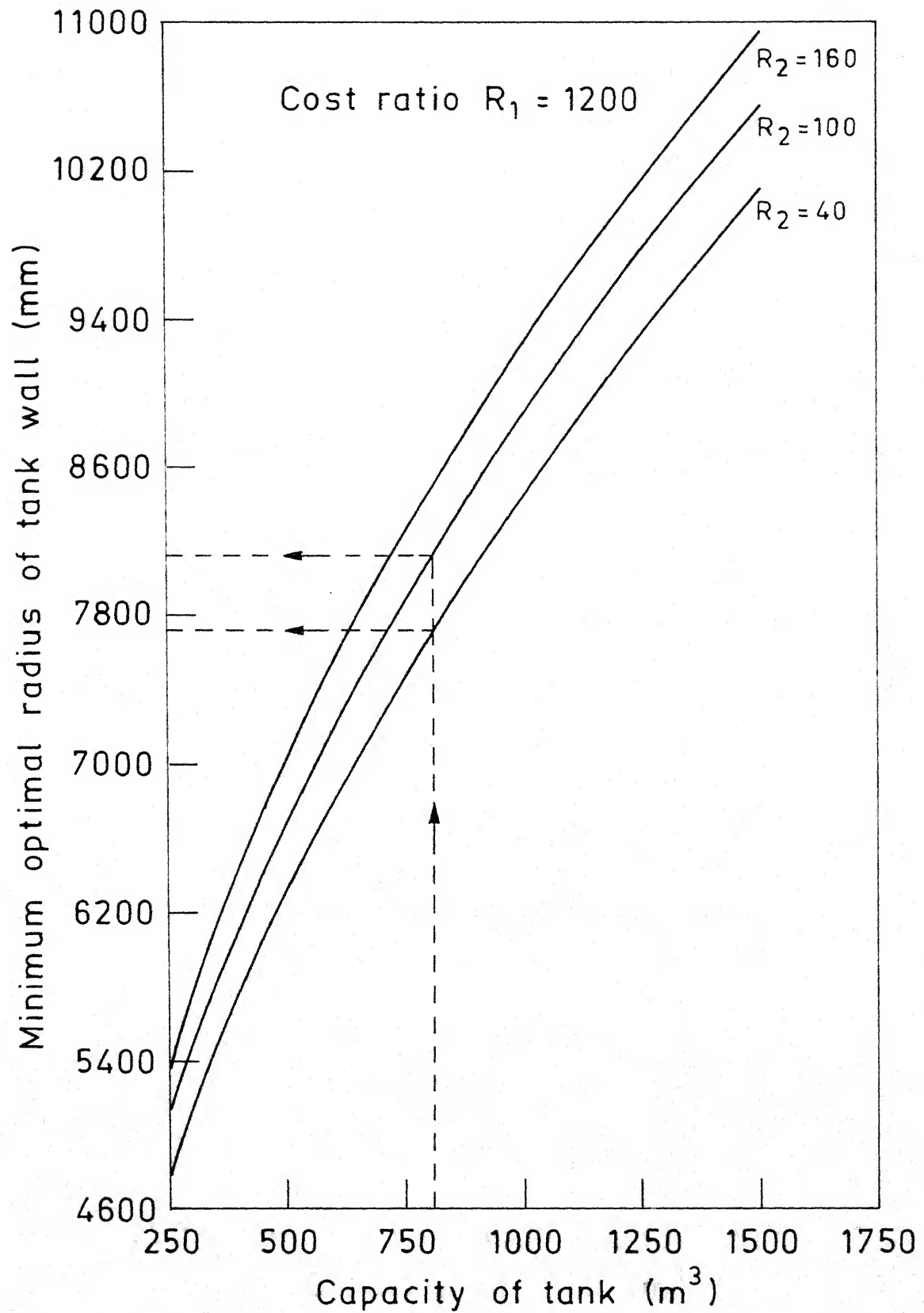Fig. 3.6 - Design chart for optimal radius of tank wall.

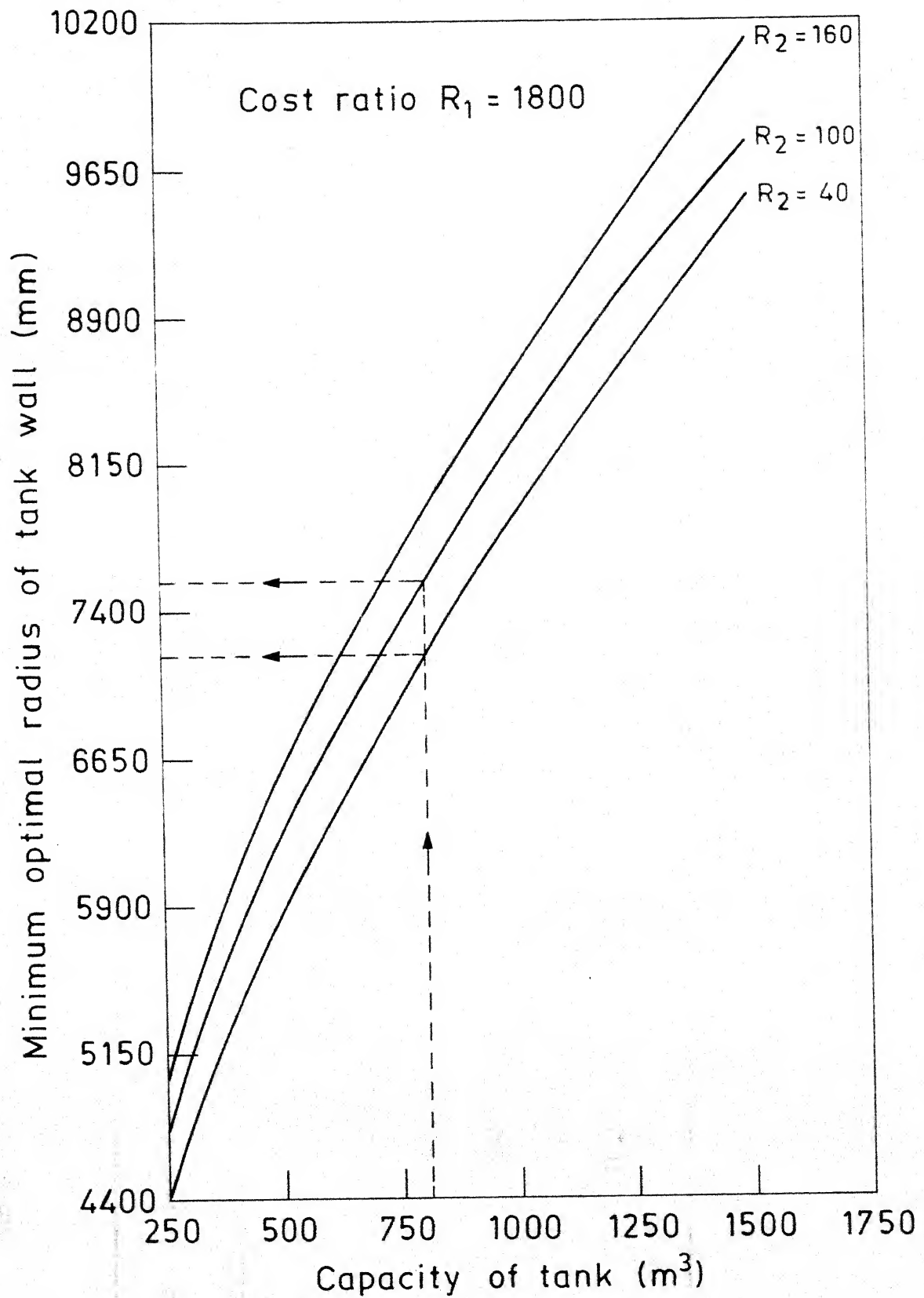Fig. 3.7 - Design chart for optimal radius of tank wall.

Fig. 3.8 - Design chart for optimal radius of tank wall.

Table 3.2   Optimal radius, in metres, for different degrees of fixity

$R_1 = 1200; \quad R_2 = 100$

Capacity (m³)	Degree of fixity					
	1.0	0.8	0.6	0.4	0.2	0.0
250	5.055	5.050	5.048	5.049	5.052	5.053
500	5.730	6.650	6.647	6.649	6.649	6.651
750	8.005	7.952	7.851	7.855	7.854	7.858
1000	8.995	8.989	8.887	8.861	8.860	8.870
1250	9.950	9.907	9.835	9.745	9.744	9.754
1500	10.759	10.717	10.702	10.541	10.539	10.549

factors, only an optimization technique enables in arriving at the optimal radius depending on the cost ratios.

The thickness of the wall has assumed the minimum value in all cases. In optimization studies, when such a situation is encountered, it is a normal practice to explore whether the specified minimum can be further reduced. In the present case, since a minimum nominal cover of 40 mm has to be provided and the value of 125 mm was arrived after considering the limitations imposed by the construction aspects, it is not desirable to reduce the specified minimum thickness any further.

The Figures 3.9 through 3.17 give the design charts for optimal areas of reinforcement a_{v2} and a_{h2} for various cost ratio combinations and different degrees of fixity. In all these cases, a_{h2} is larger than a_{v2}. This is not surprising when the permissible stresses for the reinforcement in flexure and axial tension are examined. While the maximum direct tensile stress has been limited to 130 MPa from the 'deemed to satisfy' criterion for resistance to cracking, no such upper limit has been explicitly stated for flexural tension. Flexural cracking is controlled by specifying an upper limit to crack width which is given by Eq. (3.1). It has been found that at service loads, for designs satisfying the crack width criterion, the flexural tensile stress in the reinforcement could be as high as 275 MPa. Further
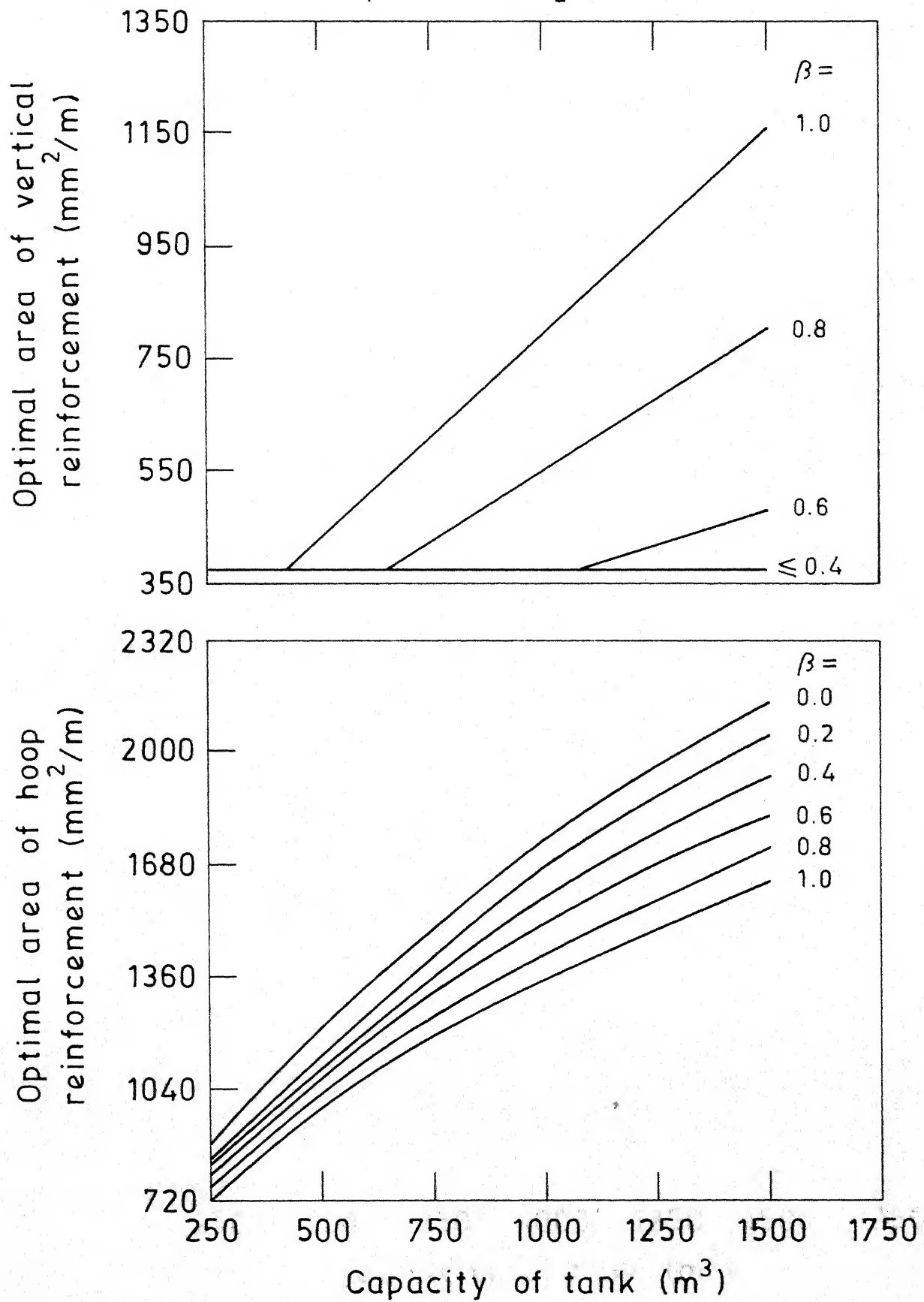
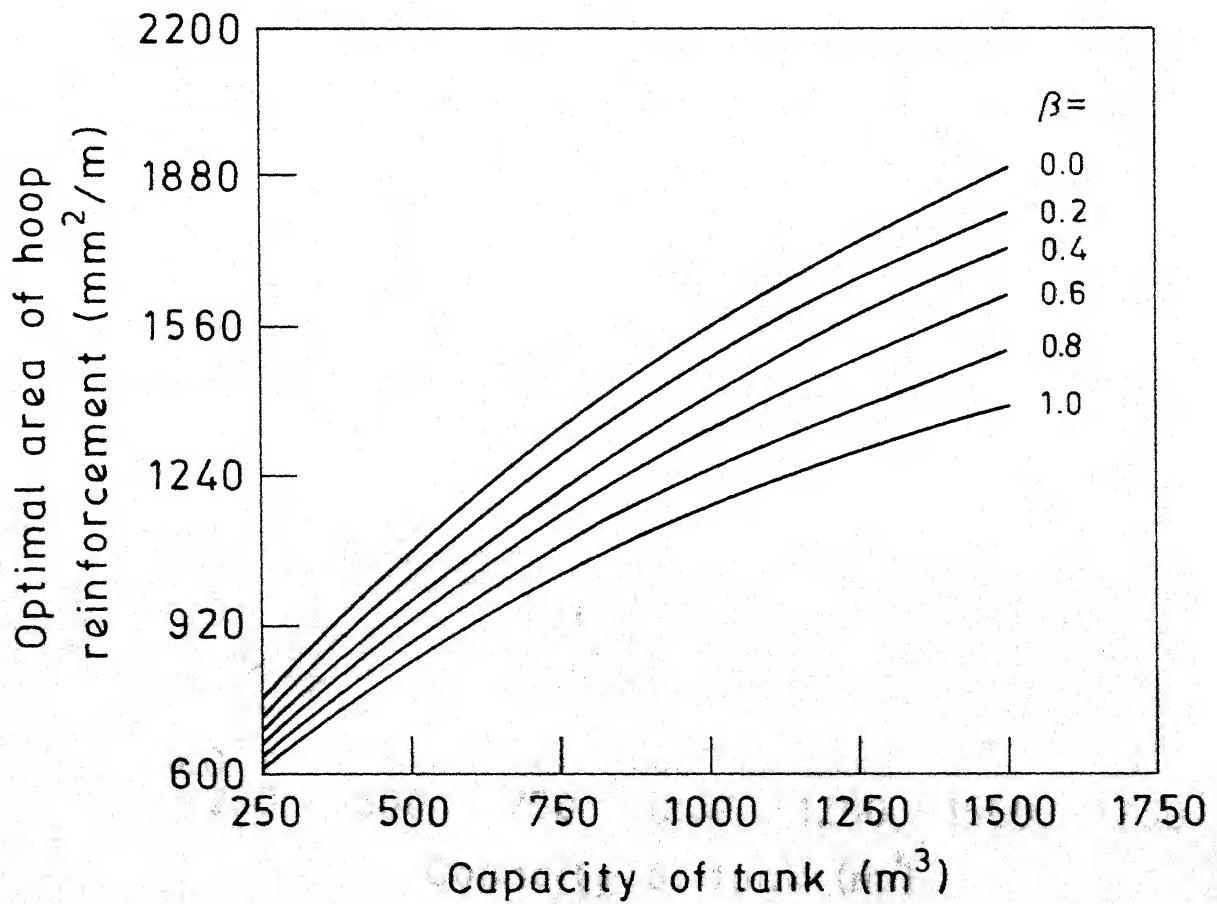Fig. 3.9 - Design chart for optimal areas of hoop and vertical reinforcements.
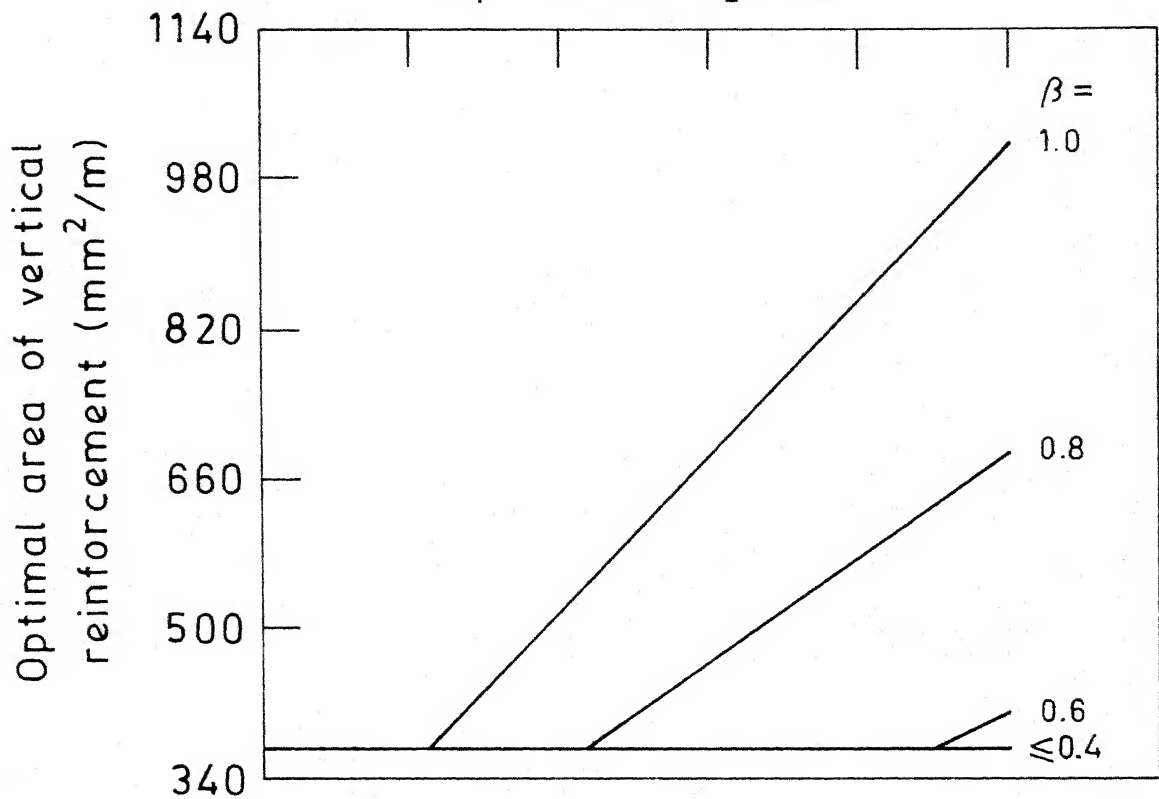
Fig. 3.10 - Design chart for optimal areas of hoop and vertical reinforcements.

Fig. 3.11 - Design chart for optimal areas of hoop and vertical reinforcements.

Fig. 3.12 - Design chart for optimal areas of hoop and vertical reinforcements.

$$R_1 = 1200 \;,\; R_2 = 100$$

Fig. 3.13 - Design chart for optimal areas of hoop and vertical reinforcements.

$R_1 = 1200$, $R_2 = 160$

Fig. 3.14 - Design chart for optimal areas of hoop and vertical reinforcements.

Fig. 3.15 - Design chart for optimal areas of hoop and vertical reinforcements.

R_1 = 1800 , R_2 = 100
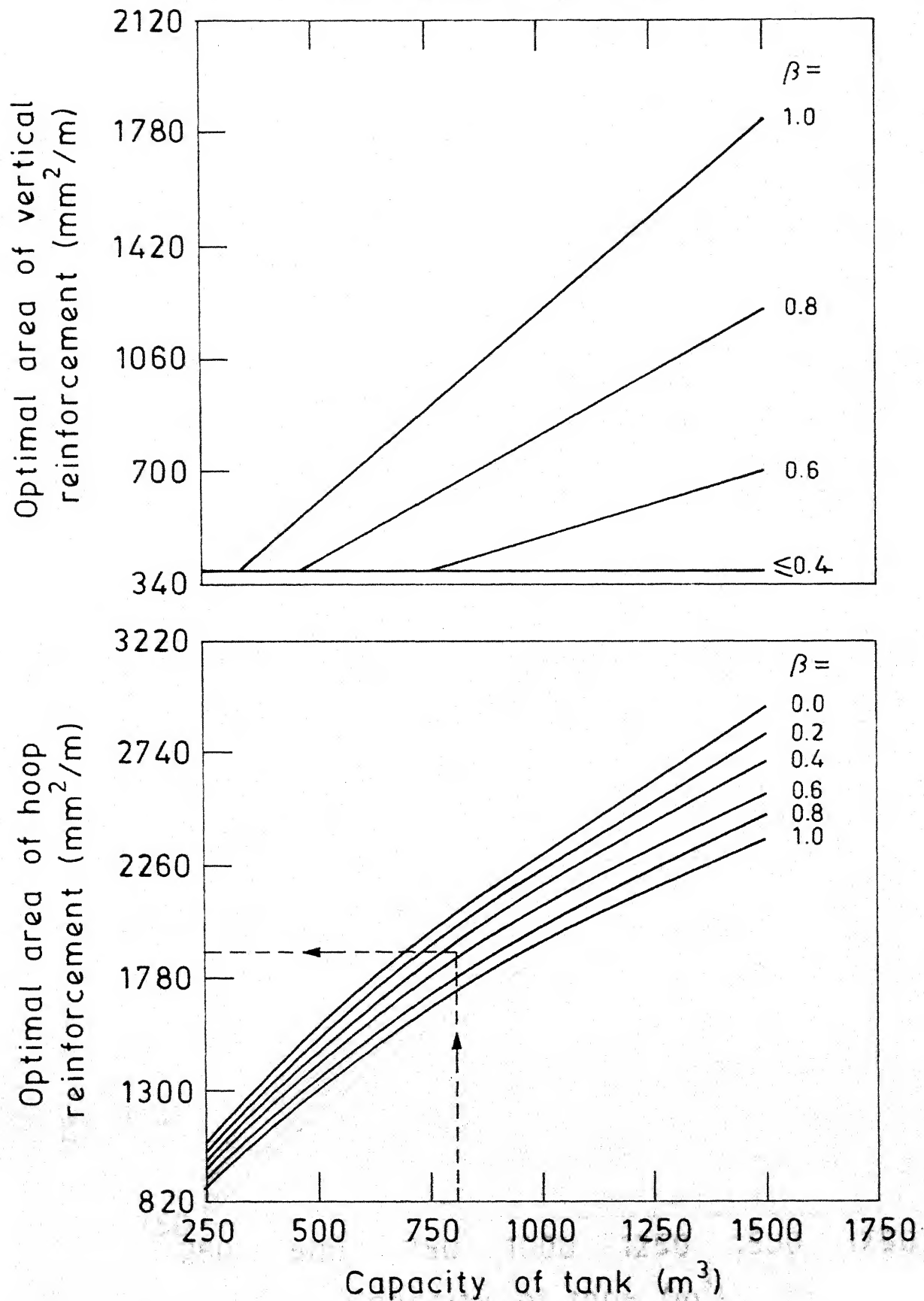
Fig. 3.16 - Design chart for optimal areas of hoop and vertical reinforcements.
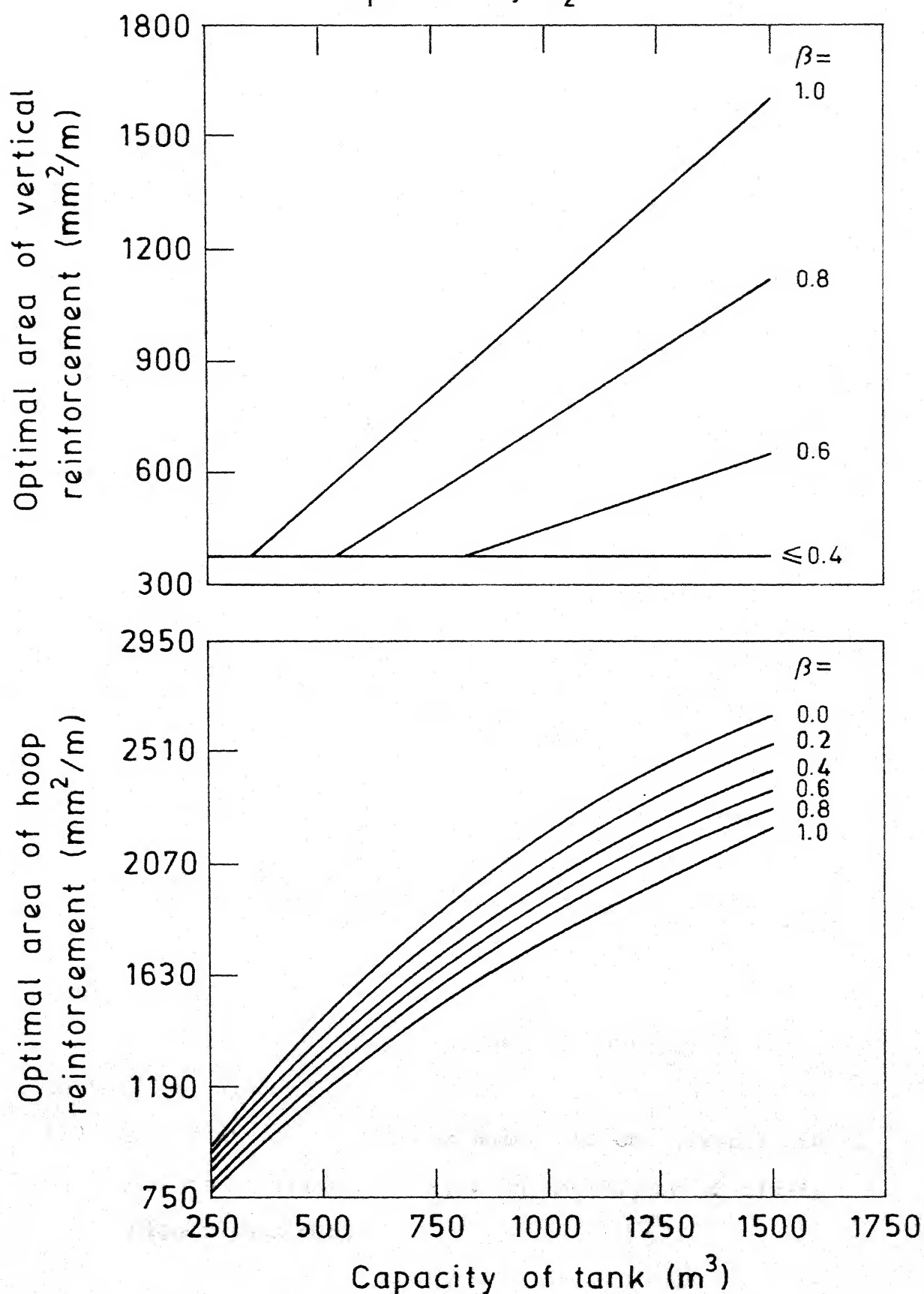
Fig. 3.17 - Design chart for optimal areas of hoop and vertical reinforcements.
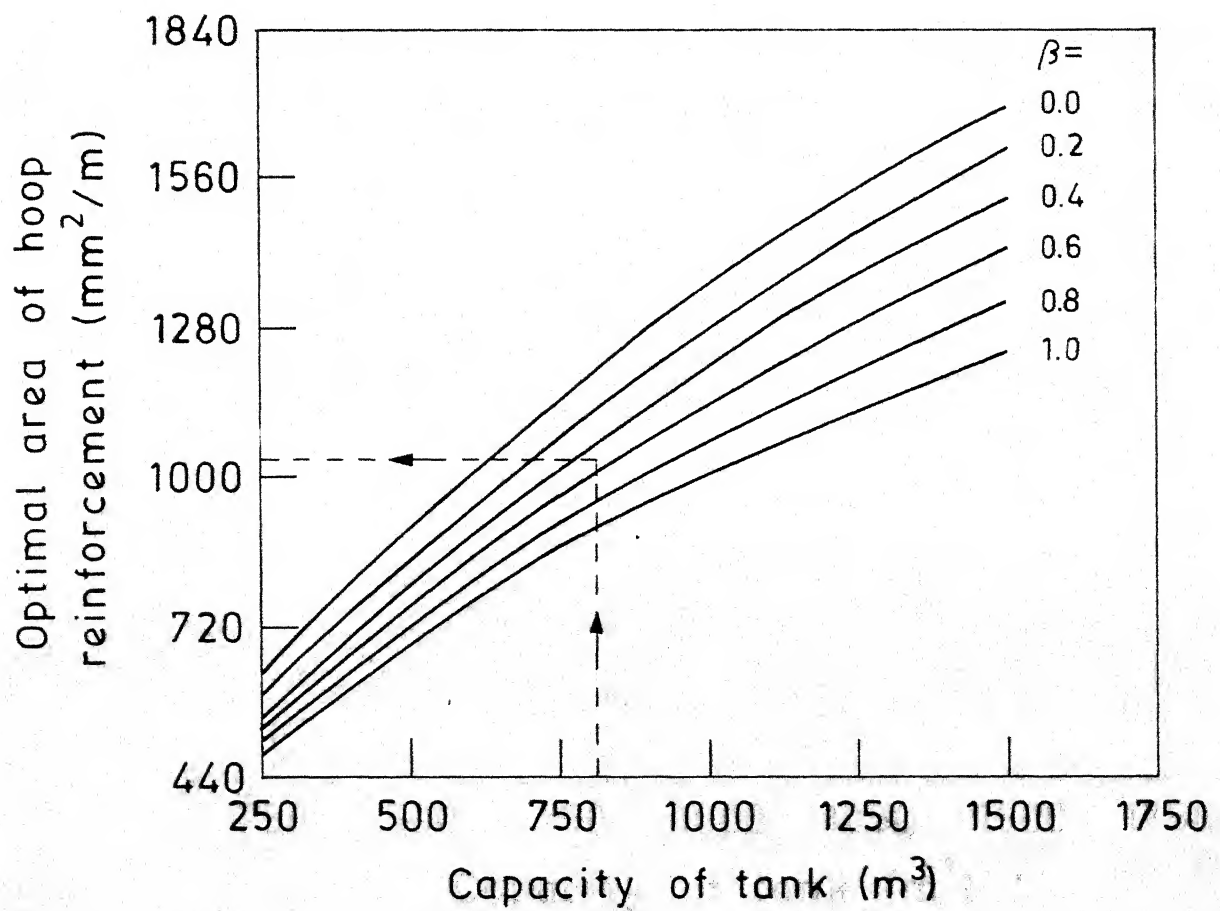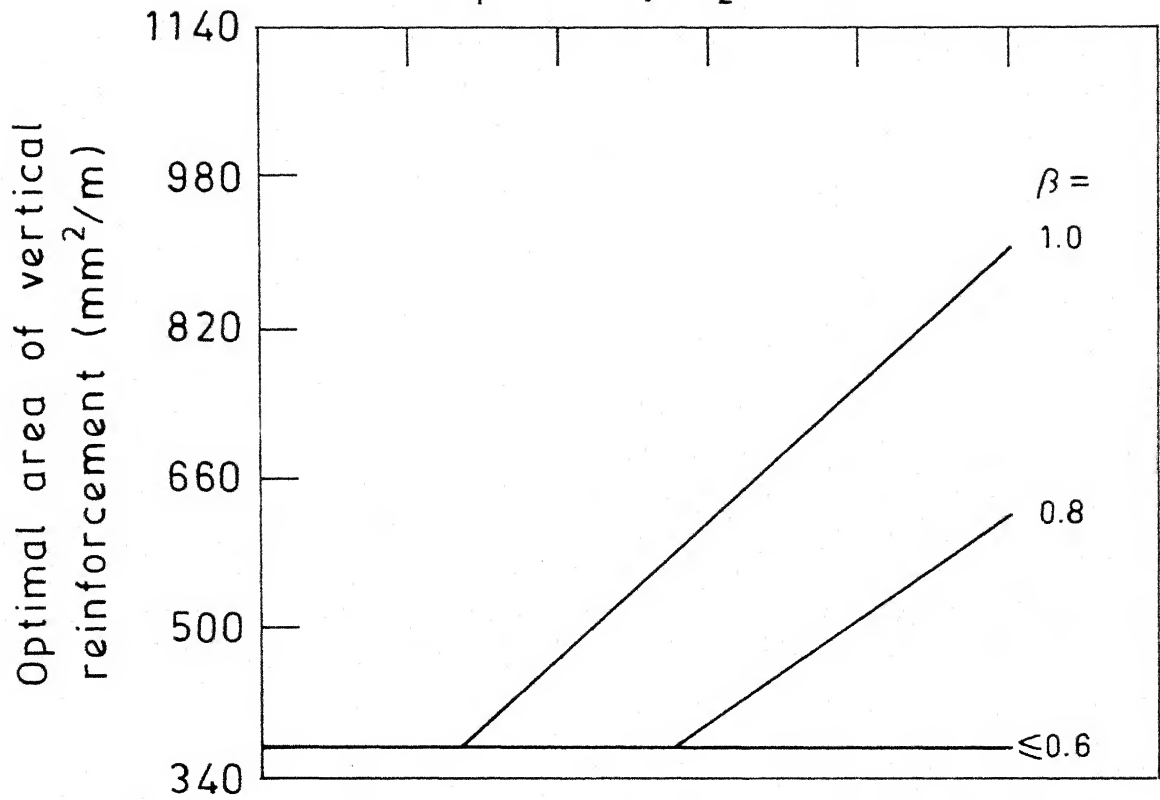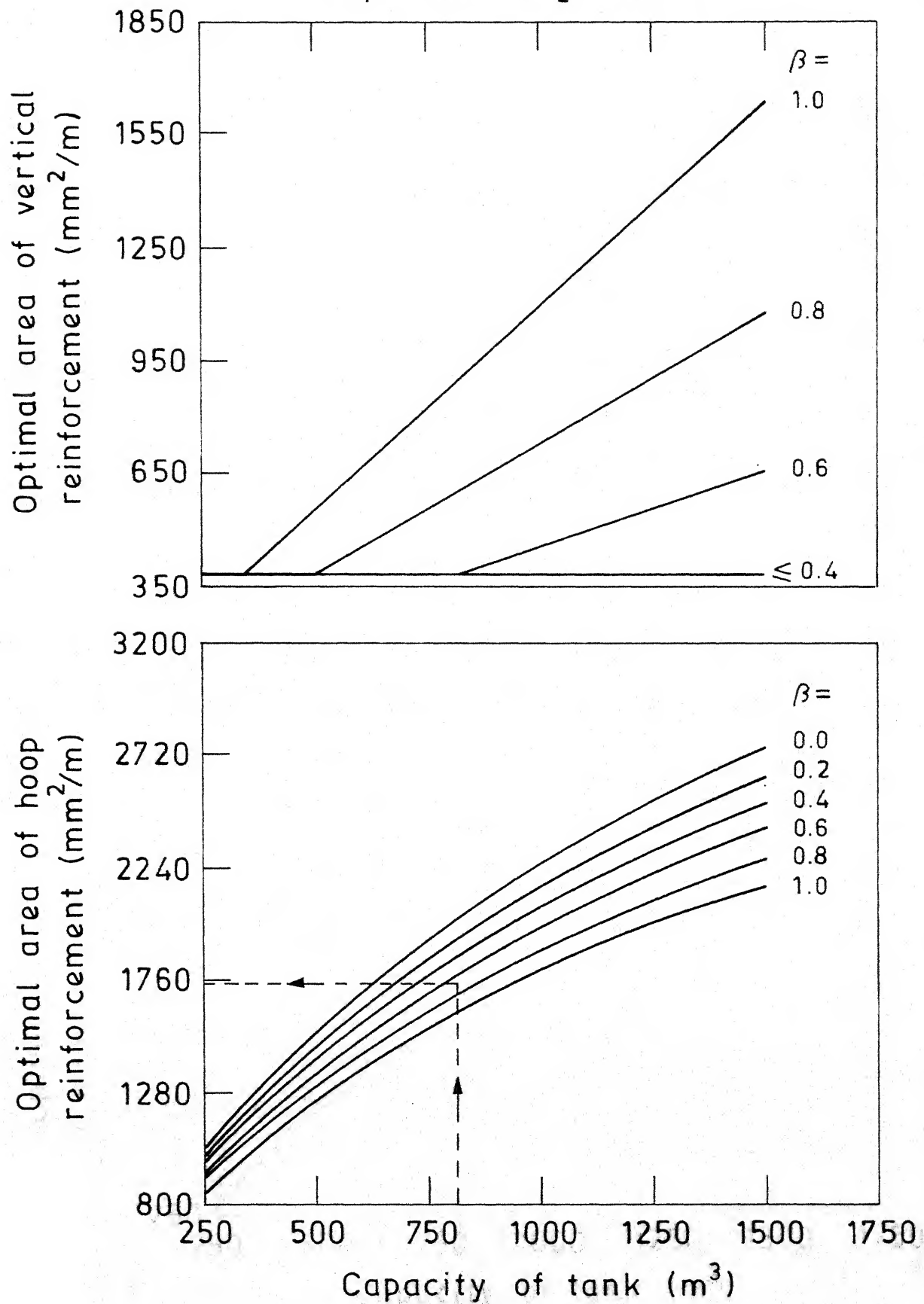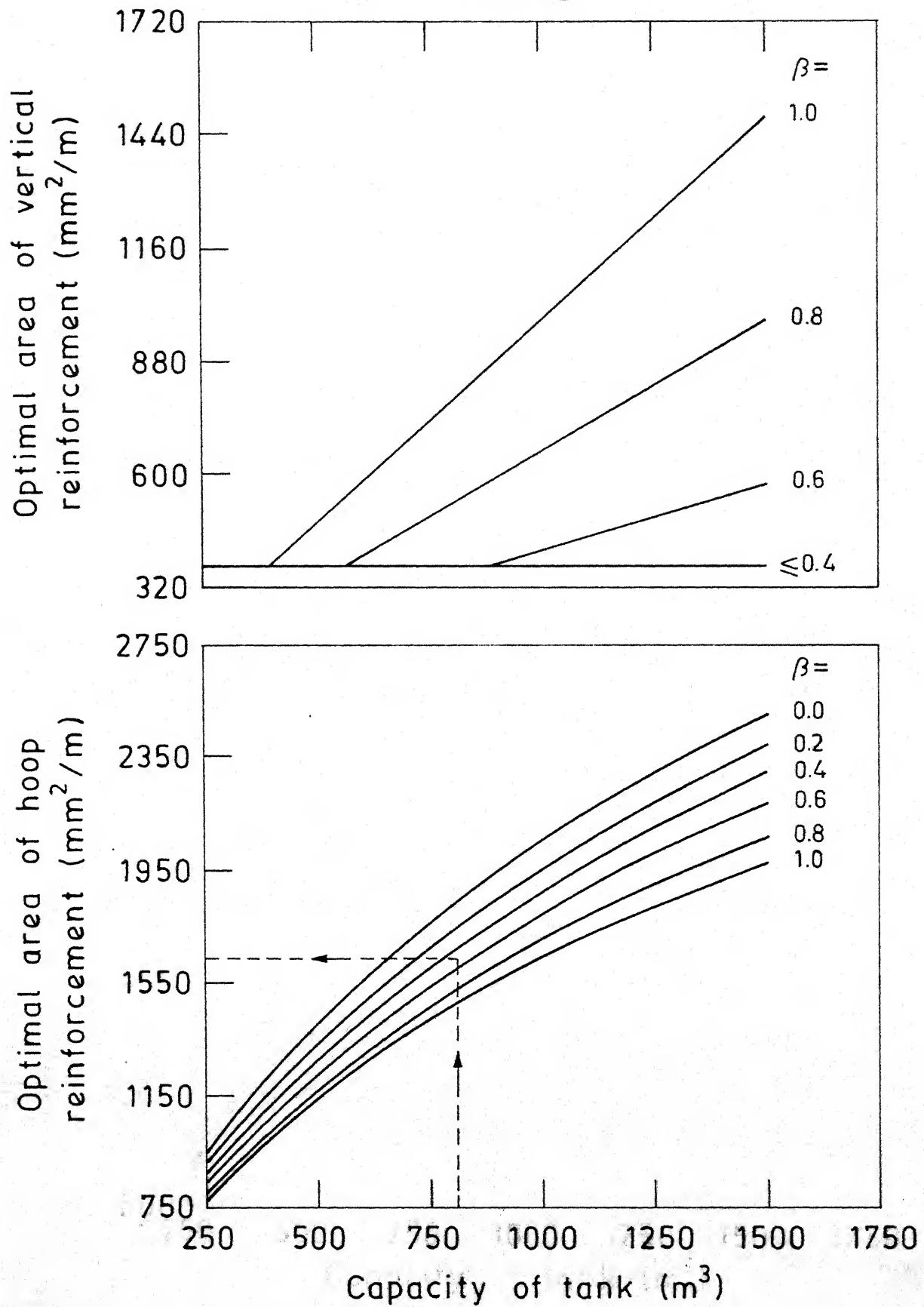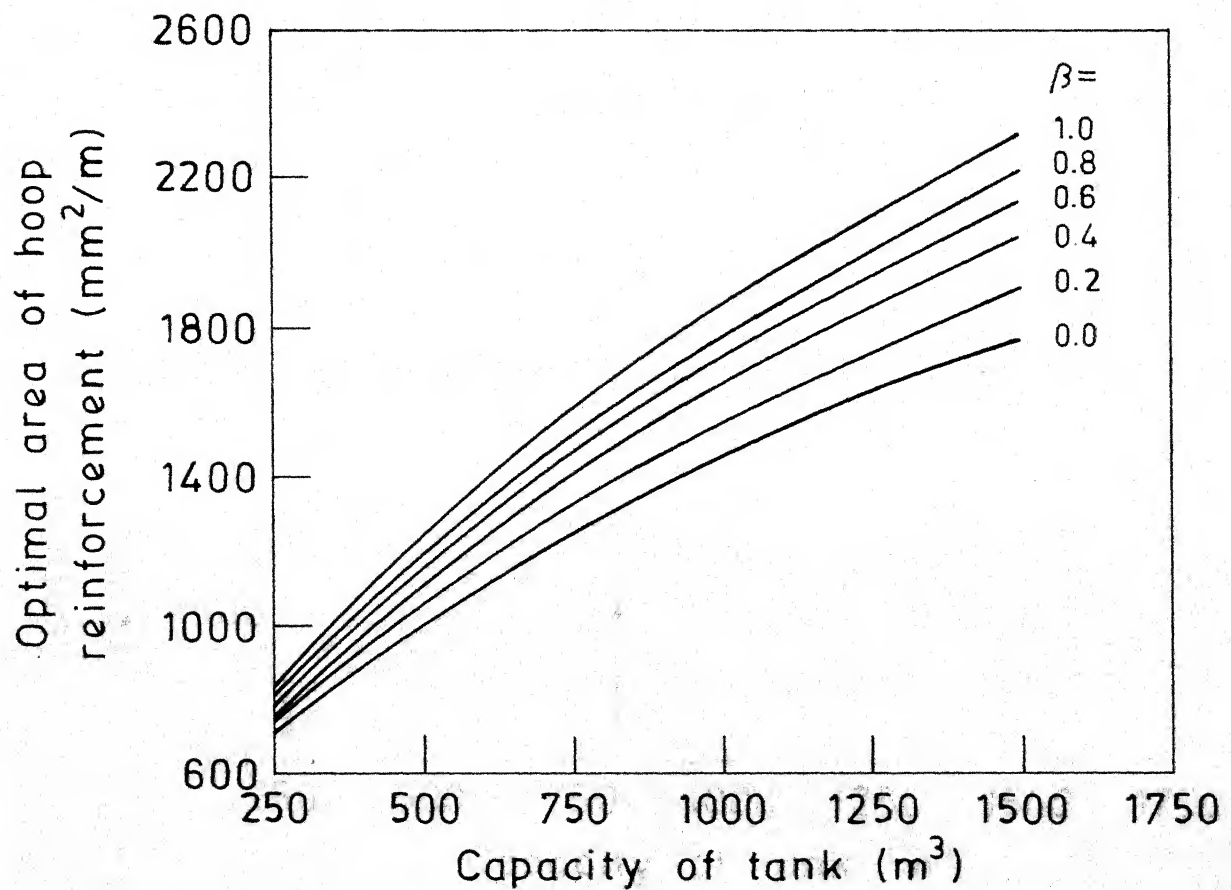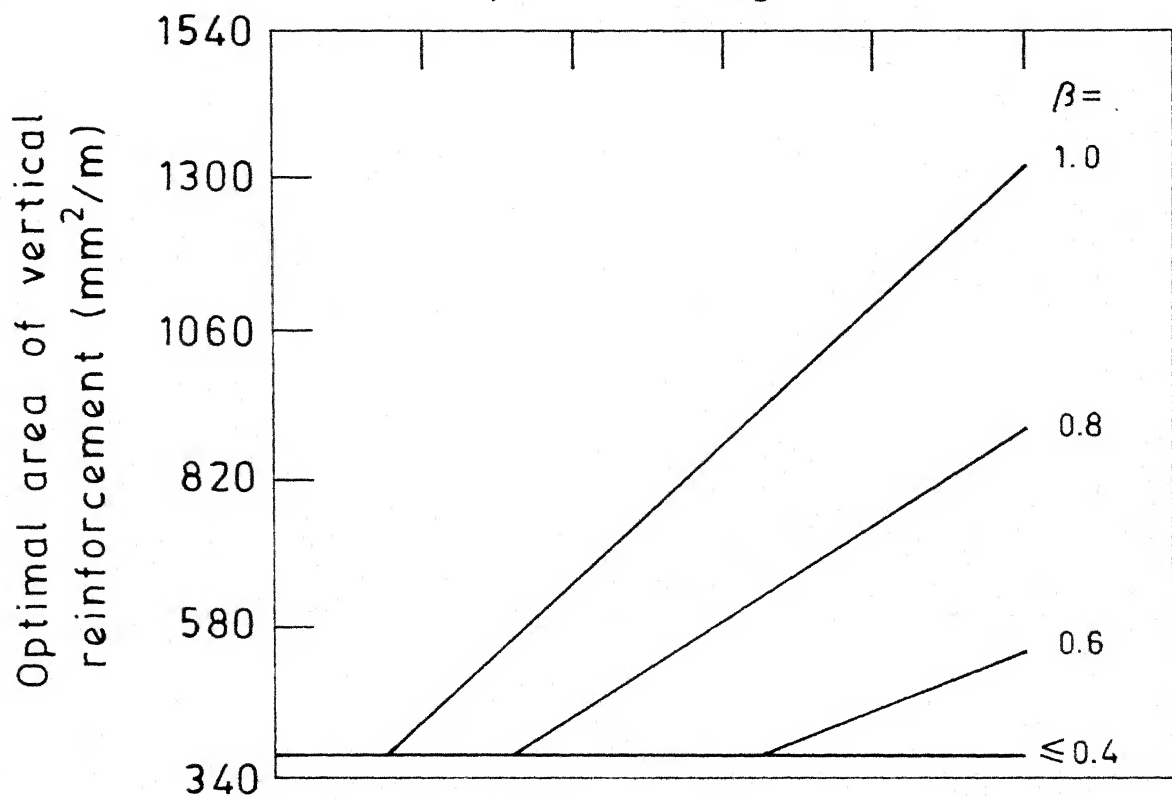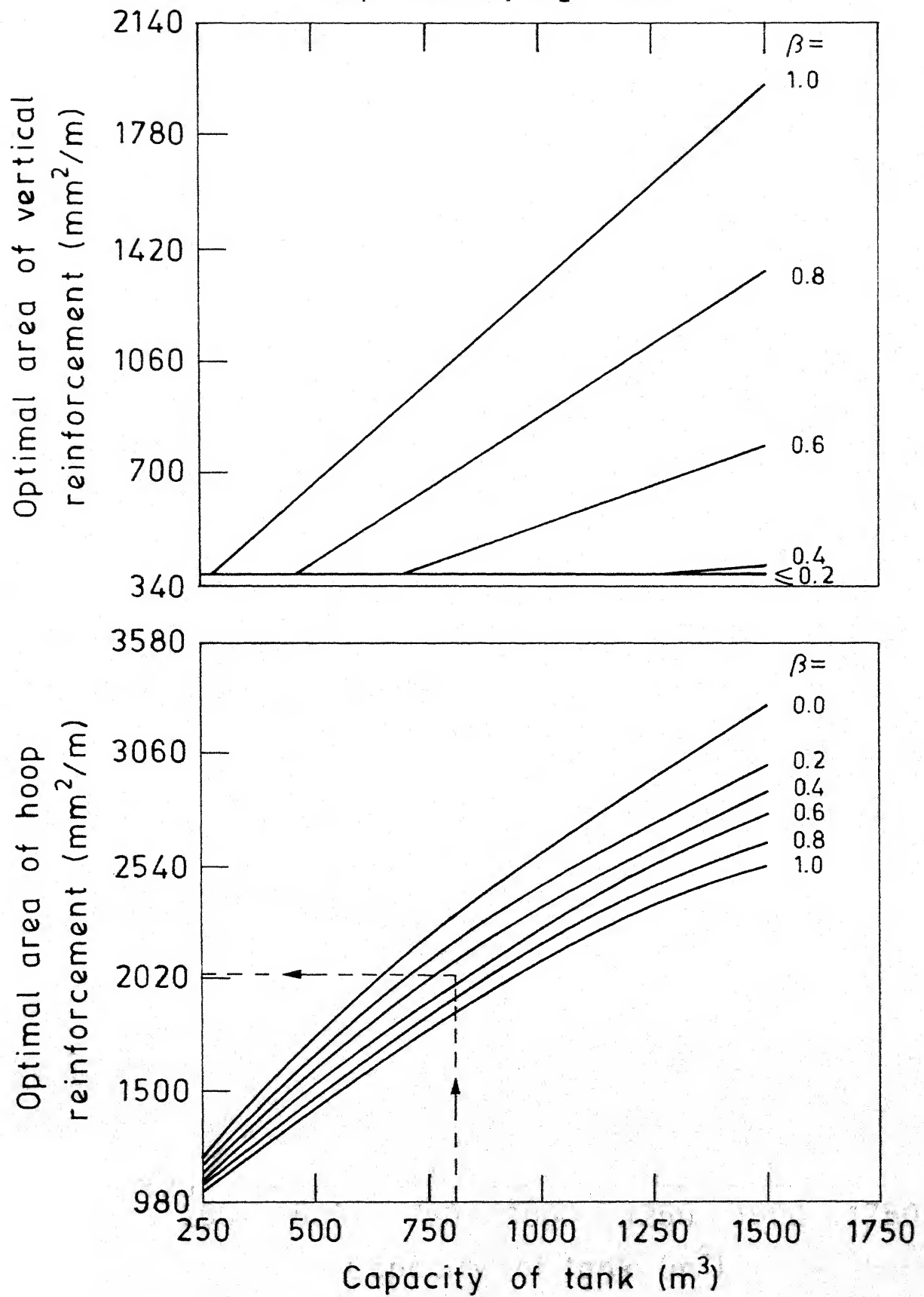
research is needed to find out whether there are possibilities
of utilizing the strength of hoop reinforcement in a better
manner.

The value of a_{v2} will be maximum when the degree of
fixity is one and goes on reducing along with decreasing
degrees of fixity till it is equal to 0.3%, beyond which value
it is not permitted to decrease. The degree of fixity at
which the minimum is reached depends on the capacity of the
tank and the cost ratio combination. The value of a_{h2}
obviously increases with the decrease in the degree of fixity.

The Figures 3.18 through 3.20 show the effect of
degree of fixity on the objective function. This again depends
on the capacity of the tank and the cost ratios. The variation
in the value of the objective function due to different
degrees of fixity is of the order of 1% to 3% only. For
smaller capacities, full degree of fixity is desirable. This
is true for larger capacities also when $R_1 = 600$. However,
for higher values of R_1, as the capacity increases, the degree
of fixity at which the objective function has a minimum value
moves towards a value of 0.5.

The following constraints are critically satisfied
in the optimal design:

(1) g_1, controlling the maximum width of flexural cracks
     due to positive moment, when the degree of fixity is
     higher than 0.5.
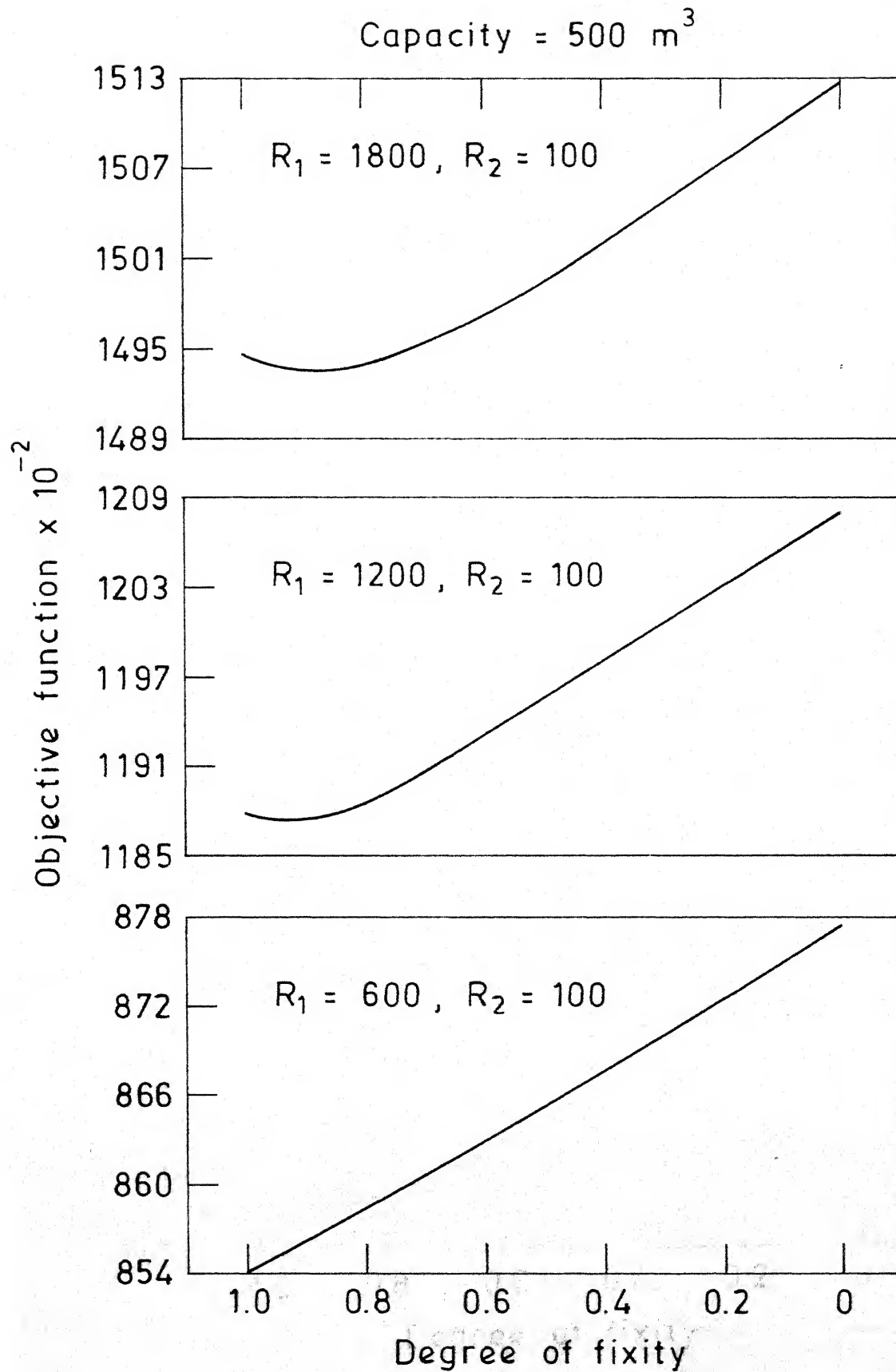
Fig. 3.18 - Effect of degree of fixity on objective function.
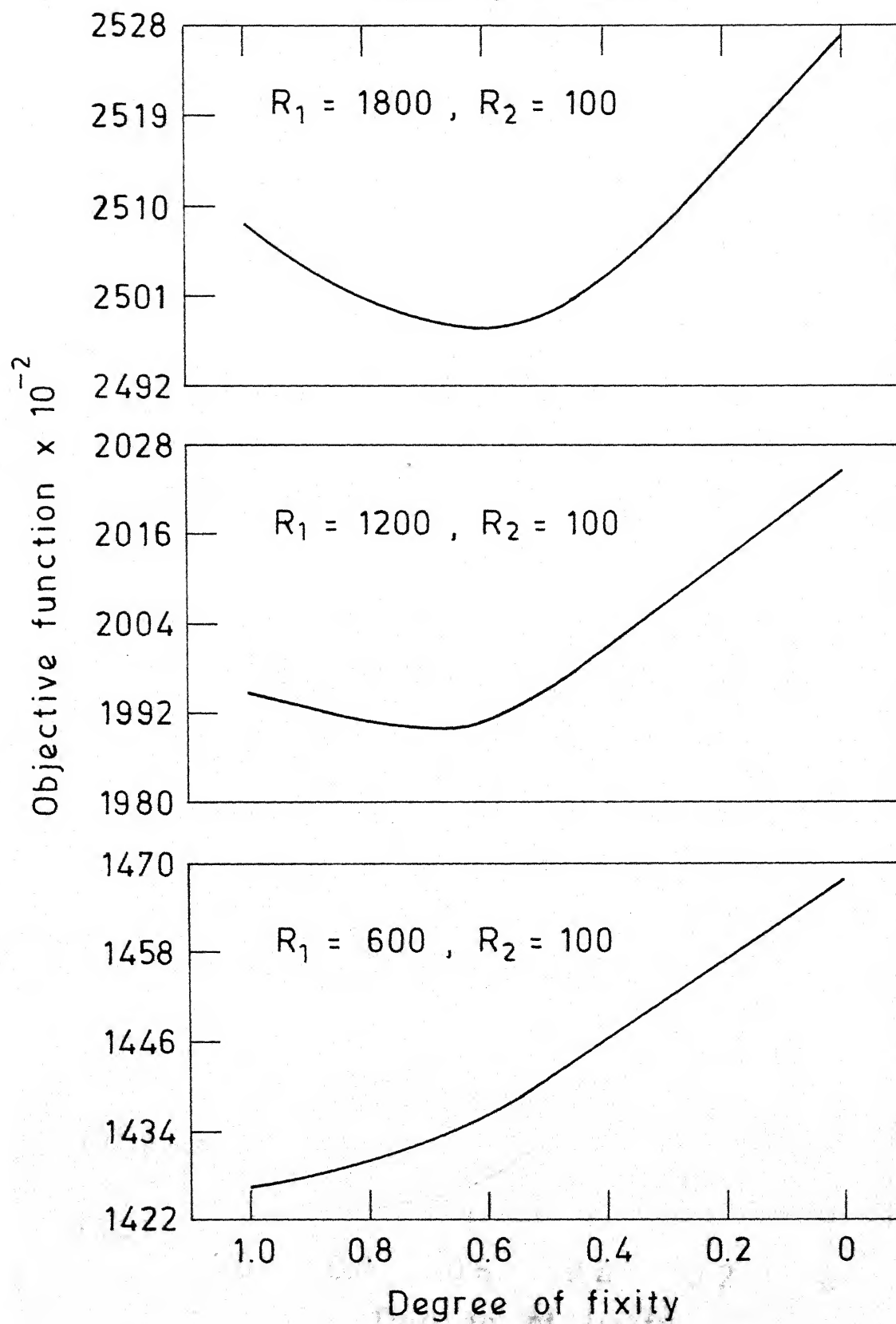
Fig. 3.19 - Effect of degree of fixity on objective function.
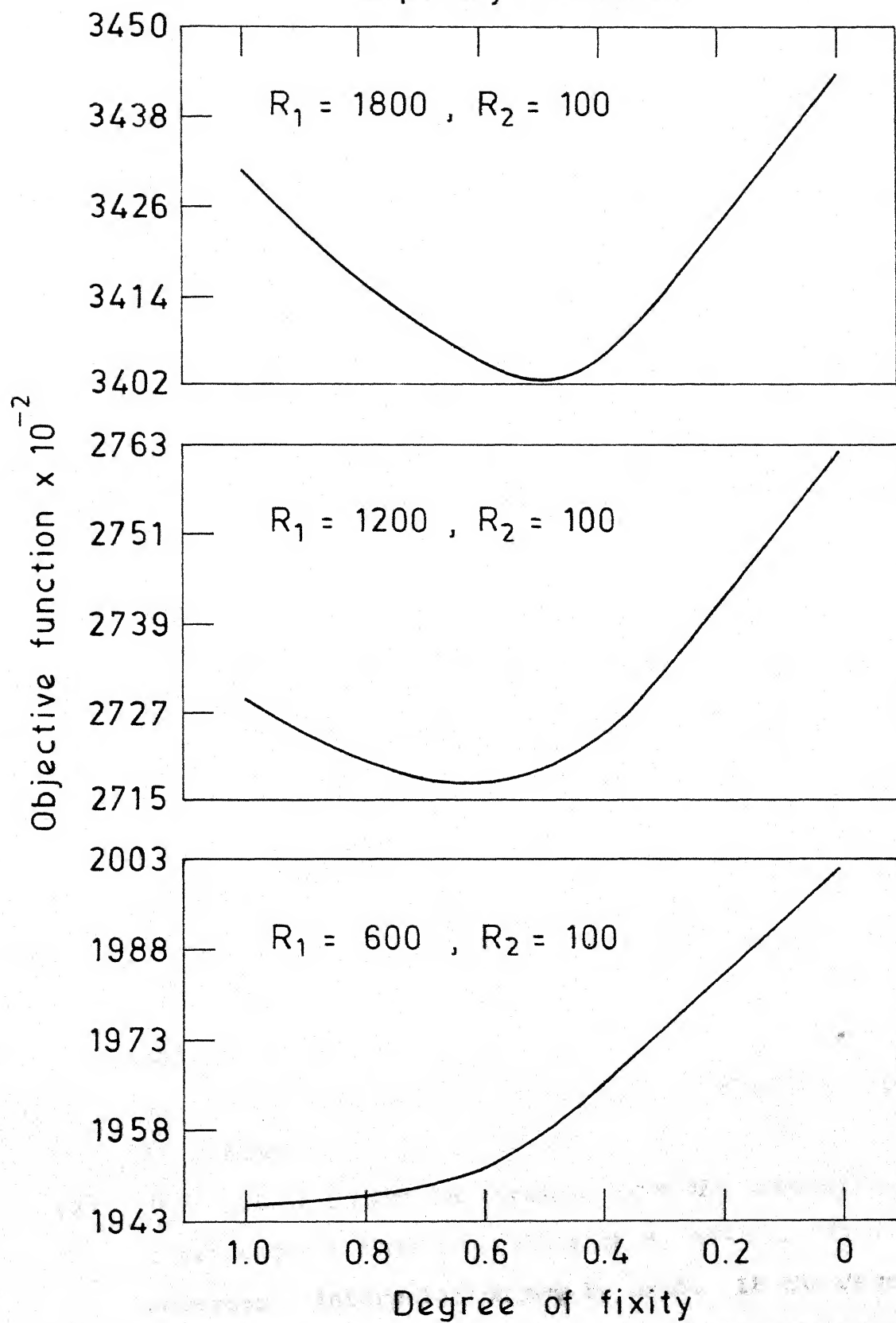
Fig. 3.20 - Effect of degree of fixity on objective function.

(2) g_7, corresponding to minimum value for a_{v2}, in other cases,

(3) g_3, dealing with the maximum value of stress in hoop reinforcement,

(4) g_5, corresponding to the minimum thickness of wall, and

(5) g_6, specifying minimum value for a_{v1}.

The collapse mechanism is found to be invariably the partial collapse of the bottom portion of the tank and the strength found to be more than adequate.

3.9   Method of Using the Design Charts

3.9.1   Procedure

The following sequence may be adopted in arriving at an optimal design of any surface water tank which is open at the top and which has a design capacity in the range 250 m^3 – 1500 m^3:

(1)   For the prevailing costs, cost ratios R_1 and R_2 are computed. From the known soil properties at the site and other conditions, if it is possible, the degree of fixity is estimated. Otherwise, a suitable value is assumed.

(2)   The optimal radius is obtained from the appropriate chart for the computed values of R_1 and R_2. When necessary, interpolation may be used. If the degree of fixity is more than 0.4, value of optimal radius

obtained from the chart may be marginally increased.

(3)   The thickness of wall is chosen to have the minimum value, with due consideration to the relevant aspects. A value in the range 125 mm - 150 mm is recommended.

(4)   Although design charts have been given to find optimal values of a_{h2} and a_{v2}, it is preferable to carry out an elastic analysis using the optimal radius, chosen thickness and estimated degree of fixity. From the bending moment diagram and hoop tension diagram, values of a_{v2}, a_{h2}, H_1, H_2, H_3, l_1 and l_2 can be easily obtained. The design chart in respect of a_{v2} and a_{h2} may be used for comparison purposes.

3.9.2   Example

The method of using the design charts is illustrated by the following example.

Data: A cylindrical surface water tank, open at the top, is required to be designed for a capacity of 825 m^3. The estimated degree of fixity is 0.5. Cost ratios are, R_1 = 1500 and R_2 = 80.

Since the two cost ratios as well as degree of fixity are different from the values considered in this study, interpolation has to be resorted to. Using the values of radius obtained from Figures 3.7 and 3.8 for cost ratios of R_1 = 1200, 1800 and R_2 = 40, 100, the value of optimal radius is interpolated as 7700 mm. In order to take

CHAPTER 4

OPTIMAL DESIGN OF SURFACE WATER TANKS WITH ROOF

4.1   Design Considerations

In order to avoid evaporation losses and pollution,
it is a common practice to provide 'a roof over water tanks.
For cylindrical tanks, such a roof may be either a slab,
with or without beams, or a spherical dome. Notwithstanding
its additional shuttering costs, a domed roof will be more
economical than a flat roof, specially for large diameters,
because of its superior structural form.

The vertical component of the inclined reaction
from the dome can be easily transmitted through the tank
wall. Although it is possible to provide the required
horizontal reaction by building the dome into the wall, such
an arrangement results in high tension in the edge regions
of the wall which often requires heavy reinforcement and an
increased thickness. Moreover, in liquid retaining structures,
it is not desirable to have large regions of high hoop
tension as they may give rise to wide cracks. A ring beam
provided at the junction of the wall and dome eliminates
all such undesirable effects and can be easily reinforced
than lengths of wall and dome which would otherwise carry
the tension.

4.2   Elastic Analysis

The Figure 4.1 shows the arrangement of the various members of a cylindrical tank with a domed roof. In this figure, R_d and t_d are the radius and thickness of the dome; the ones without any subscript refer to those of the wall. In order to find out the forces in the component members of the system due to their interaction when subjected to external forces, the analysis is carried out in two stages. First, each member is analysed independently under the action of external forces. The resulting deformations are shown by dotted lines in Figure 4.2. In the second stage, joint forces which are required to bring about compatibility of deformations are determined. The net forces in any member will obviously be the algebraic sum of those caused by the two effects.

If a spherical dome is simply supported along its periphery and subjected to a uniform load of intensity q_d over its plan area, then the non-vanishing stress resultants will be meridional and hoop membrane forces N_m and N_h only. At any angle $\emptyset$ from the vertical, these are given by

$$N_m = \frac{q_d R_d}{1 + \cos\emptyset} \, , \qquad\qquad ..(4.1)$$

$$\text{and} \qquad N_h = q_d R_d (\cos\emptyset - \frac{1}{1 + \cos\emptyset}) \, . \qquad ..(4.2)$$
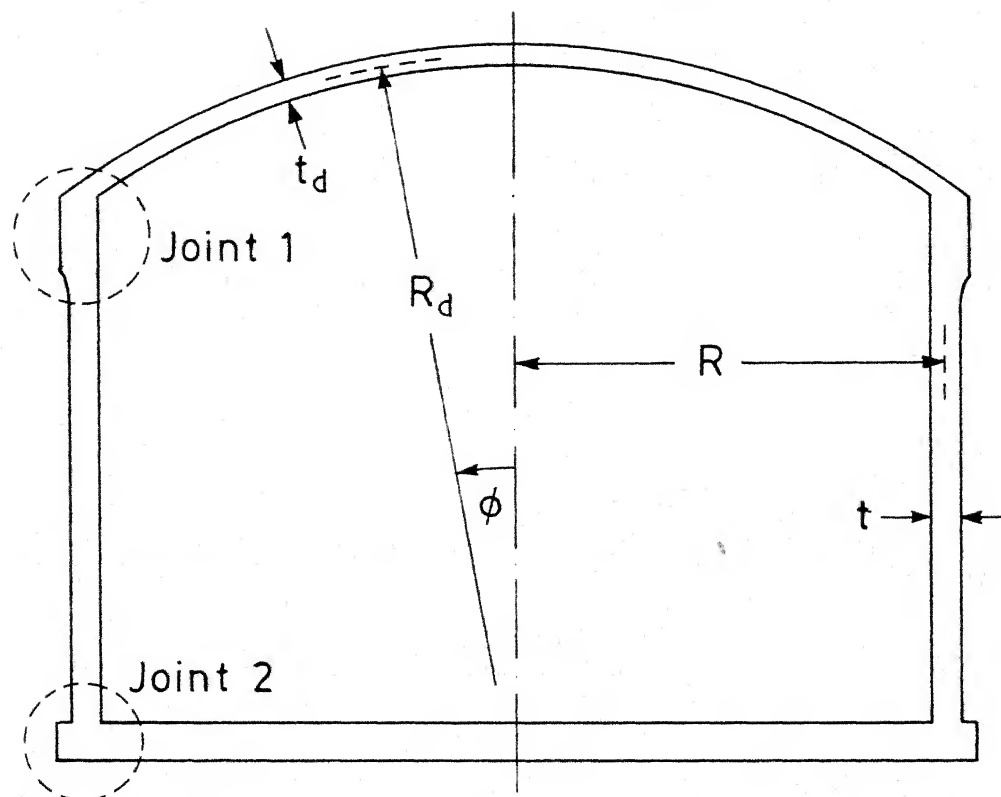
Fig. 4.1 - Cylindrical surface tank with a domed roof.
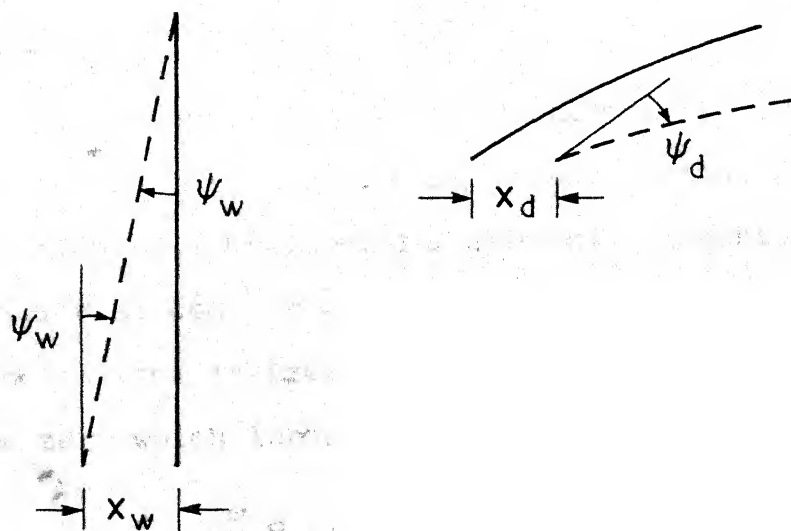
Fig. 4.2 - Free displacements of the wall and dome.

The displacements of the dome edge consisting of a clockwise rotation ψ_d and an inward translation x_d at joint 1 are given by

$$\psi_d = \frac{2q_d R_d \sin\phi_o}{E t_d} \, , \qquad\qquad ..(4.3)$$

and

$$x_d = \frac{q_d R_d^2}{E t_d} (\cos\phi_o - \frac{1}{1 + \cos\phi_o}) \, , \qquad ..(4.4)$$

where ϕ_o is the semi-central angle of the dome.

The tank wall, under the action of water pressure, suffers a clockwise rotation ψ_w at joints 1 and 2 and an outward horizontal translation x_w at joint 2 which can be determined from the following equations:

$$\psi_w = \frac{\gamma_w R^2}{E t} \, , \qquad\qquad ..(4.5)$$

and

$$x_w = \frac{\gamma_w R^2 H}{E t} \, . \qquad\qquad ..(4.6)$$

For compatibility of deformations, it is necessary to cause an inward translation x_w at the bottom of the wall by application of a horizontal force and a moment. These forces will also cause a clockwise rotation θ_w. If the bottom of the tank is treated as hinged, then the moment should be zero which leads to the relationship

$$k_{mm}^w \theta_w + k_{mh}^w x_w = 0 \, , \qquad\qquad ..(4.7)$$

where the stiffness factors associated with the wall are given by $k_{hh}^W = 4 \mu^3 D_W$, $k_{hm}^W = k_{mh}^W = 2 \mu^2 D_W$, $k_{mm}^W = 2 \mu D_W$, and $D_W = Et^3/12$. Symbols μ and E have the same meaning as given in Section 3.2. But, the flexural rigidity of the wall, EI, is denoted here by D_W for convenience. Solving for the value of Θ_W from Eq. (4.7), the net anticlockwise rotation of the bottom edge of the wall, Θ_2, is given by

$$\Theta_2 = -\Theta_W - \psi_W = \frac{\gamma_W R^2}{Et} (\mu H - 1) . \qquad ..(4.8)$$

A similar procedure is adopted to consider the effect of compatibility at joint 1. If the final displacements at this joint are an inward translation x_1 and an anticlockwise rotation Θ_1, these can be obtained from the solution of the following joint equilibrium equations:

$$k_{mm}^d (\Theta_1 + \psi_d) - k_{mh}^d (x_1 - x_d) + k_{mm}^b \Theta_1 + k_{mm}^W (\Theta_1 + \psi_W) + k_{mh}^W x_1 = 0,$$
$$..(4.9)$$

and

$$k_{hh}^d (x_1 - x_d) - k_{hm}^d (\Theta_1 + \psi_d) + k_{hh}^b x_1 + k_{hh}^W x_1 + k_{hm}^W (\Theta_1 + \psi_W)$$

$$+ \frac{q_d R_d \cos\phi_o}{(1 + \cos\phi_o)} = 0 , \qquad ..(4.10)$$

where the stiffness terms corresponding to the beam and dome and some associated parameters are given by

$$k_{mm}^{b} = Eh_{b}^{3}/12R^{2}, \quad k_{hh}^{b} = Ebh_{b}/R^{2}, \quad k_{mm}^{d} = \frac{R_{d}Et_{d}}{4\lambda^{3}}\left(f + \frac{1}{f}\right),$$

$$k_{mh}^{d} = k_{hm}^{d} = \frac{Et_{d}}{2\lambda^{2}f\,\sin\phi_{o}}, \quad k_{hh}^{d} = \frac{Et_{d}}{\lambda R_{d}f\,\sin^{2}\phi_{o}}, \quad \lambda = 3\left(\frac{R_{d}}{t_{d}}\right)^{2},$$

$f = 1 - \frac{1}{2\lambda}\cot\phi_{o}$, b = width of ring beam and h_{b} = depth of ring beam. The last term in Eq. (4.10) represents the horizontal component of the meridional thrust at the dome edge.

These steps enable in determining the net displacements of joints 1 and 2 from which the forces on the dome and ring beam can be easily determined. In order to determine the moments and hoop tensions in the tank wall, radial displacement, w, of the tank wall given by Eq. (3.8) is considered. The following boundary conditions are used in order to determine the four constants of integration:

$$w\Big|_{x=0} = x_{1}, \qquad\qquad ..(4.11)$$

$$\frac{dw}{dx}\Big|_{x=0} = \theta_{1}, \qquad\qquad ..(4.12)$$

$$w\Big|_{x=1} = 0, \qquad\qquad ..(4.13)$$

$$\frac{dw}{dx}\Big|_{x=1} = (1-\beta)\theta_{2}, \qquad\qquad ..(4.14)$$

where β denotes the degree of fixity at the base of the tank wall. The solution of the resulting set of 4 linear simultaneous equations enables in defining the deflection configuration. The other steps like determining the bending

moment and hoop tension distribution as well as their maximum values follow the same pattern as discussed for tanks without roof.

4.3   Limit Analysis

For a given capacity of the tank, it can be intui-tively guessed that the optimal radius of a tank with a roof will be less than that without one, in view of the cost of dome also coming into picture for the former. This results in a comparatively longer shell and therefore failure of the shell by partial collapse discussed in Section 3.3.6 is expected to be valid for this type of tanks as well.

4.4   Design of Dome and Ring Beam

For economy, the rise of dome should be in the range of 0.25 R to 0.4 R and in order to avoid necessity of formwork for the top surface, the semi-central angle should not exceed $40°$. Based on these considerations a value of $30°$ is chosen for $\emptyset_0$. The domed roof is designed for a service load of 3.2 kN/m^2. It is found that the membrane forces given by Eqs. (4.1) and (4.2) even for the largest diameter do not demand a thick dome. However, practical considerations preclude consideration of any thickness less than 100 mm. The area of reinforcement to be provided is also governed by the minimum value of 0.3% except near the edges, in some cases. The edge forces imposed by compatibility requirements

die out fast and extra reinforcement required at the edges, if any, can be curtailed within a short distance. Cost of the dome in the objective function is computed, thus based on a 100 mm thick dome with a semi-central angle of 30° and reinforcement of 0.3% in both directions.

Dimensions of the ring beam can be chosen as design variables; but, cost of the ring beam in relation to the cost of the system is very small. Besides, efficiency of any optimization technique will decrease with the increase in the number of design variables. As such, the ring beam is chosen to be 1.5 t wide and 2 t deep, t denoting the thickness of wall. Reinforcement required for the ring beam is obtained from the hoop tension consideration.

4.5   Optimization

4.5.1   Objective function

The objective function for such a tank can be treated as the sum of the costs of floor, wall, ring beam, and dome. This can be written in the form

$$F = F_f + F_w + F_b + F_c = F' + F_b + F_c , \quad ..(4.15)$$

where F' is the value of objective function given by Eq. (3.55) for a tank without roof. Adding the costs of ring beam and dome,

$$F = F' + R_1(2\pi Rbh_b + A_{dome}t_d) + \gamma_s(2\pi RA_h$$

$$+ 0.006\, A_{dome}t_d) + R_2 A_{dome}\, , \qquad\qquad ..(4.16)$$

where A_{dome} = surface area of the domed roof, and

$\quad\quad A_h$ = area of hoop reinforcement for the ring beam.

4.5.2   Constraints

The constraints for this tank will be essentially same as those considered for the tank without roof. The only change will be in respect of the allowable stress in the hoop reinforcement. In this case, the top portion of the tank is likely to be exposed to saturated water vapour continuously. As such, for a height H_1 from the top, class of exposure is treated as A and allowable tensile stress in the reinforcement limited to 100 MPa.

4.6   Example

Similar to the example considered in Section 3.6, indirect design of a tank for a capacity of 750 m^3 with a free board of 200 mm is obtained and compared with optimal limit state design. Degree of fixity is assumed to be 0.4 for the indirect design. Optimal design has been obtained for both $\beta = 0.4$ and for $\beta = 1.0$. Details of the indirect design are as follows:

With an assumed radius of 6 m, the thickness of wall required is found to be 210 mm. Maximum values of

forces in the tank wall are, positive bending moment = 8.872 kNm/m, negative moment = 6.12 kNm/m, and hoop tension = 320 kN/m.

Adopting a hoop reinforcement of 2470 mm^2/m in the central portion of the tank, maximum direct tensile stress in concrete is 1.31 MPa, which is just permissible. The curtailed hoop reinforcement provided in the top 2.77 m (allowable tensile stress = 100 MPa) is 1610 mm^2/m and bottom 0.54 m (allowable tensile stress = 130 MPa) is 1240 mm^2/m.

A thickness of 210 mm demands a minimum reinforcement of 630 mm^2/m in the vertical direction. This value has been found to be adequate in terms of both concrete and steel stresses due to bending moment.

The Table 4.1 gives the values of design variables for the two optimal solutions OPD1 and OPD2 for β = 0.4 and β = 1.0 respectively as well as the values of objective function. For the indirect design, value of the objective function for the different cost ratios considered is given within brackets along with that of OPD1.

Comparison of the results for tanks with and without roof for a capacity of 750 m^3, using Tables 3.1 and 4.1, indicate that for any particular cost ratio combination, optimal radius of those with roof are less than those without roof as had been guessed initially. The predicted mode of failure by partial collapse has also been found to be valid.

Table 4.1 Optimal values of design variables and objective function

$$R_2 = 100$$

Cost ratio R_1	Design nomenclature	Radius R (m)	Thickness t (mm)	Reinforcement a_{v2} (mm²/m)	Reinforcement a_{h2} (mm²/m)	Objective function
600	OPD1	7.20	125	375	1905	162040 (190228)
	OPD2	7.19	125	970	1765	160776
1200	OPD1	6.68	125	375	2190	210275 (247756)
	OPD2	6.71	125	1160	2030	209285
1800	OPD1	6.33	125	375	2395	255830 (305284)
	OPD2	6.35	125	1325	2250	255210

For the optimal designs, the thickness of the tank
wall is again seen to assume the minimum value. The percen-
tage saving in the cost due to the combined effects of optimi-
zation and limit state approach is of the order of 15% to 16%
for this example.

4.7   Parametric Studies

Optimal limit state designs have been carried out
for the following cases of a cylindrical surface tank with a
dome cover:

(1)   capacity in m^3 :   250, 500, 750, 1000, 1250 and 1500;

(2)   degree of fixity :   0.0, 0.2, 0.4, 0.6, 0.8 and 1.0;

(3)   cost ratio R_1 :   600, 1200 and 1800; and

(4)   cost ratio R_2 :   40, 100 and 160.

A free board of 200 mm has been provided in all the
cases.

4.8   Results and Discussion

For this type of tanks, it has been found that
variation of cost ratio R_2 and degree of fixity β, do not
influence the value of the optimal radius significantly.
This is again because of the fact that the optimal radius
is controlled by the cost of both floor and roof and
therefore there is a lesser degree of freedom for the radius
of the wall to increase in order to reduce cost of formwork

or reinforcement to resist moment. Effect of change in the values of R_2 and β on the optimal radius is presented in Tables 4.2 and 4.3.

From these tables it is seen that the optimal radius decreases with a decrease in the values of both R_2 and β (up to certain level). However, the variations in the values of optimal radius due to different degrees of fixity may be ignored. Effect of changes in the value of R_2, though not insignificant, does not warrant its inclusion in the design chart. The Figure 4.3 gives the design chart for choosing the optimal radius for different values of R_1, assuming the value of R_2 equal to 100. For other values of R_2, the optimal radius may be marginally adjusted, if found necessary.

Optimal thickness of the tank wall, for all the cases considered, is again found to be 125 mm. The Figures 4.4 through 4.6 give the design charts to find the optimal values of a_{v2} and a_{h2} for different values of R_1. Effect of degree of fixity on the objective function is shown in Figures 4.7 through 4.9. The discussion on these aspects as well as those regarding the constraints which are active, given in Section 3.8, is valid for this type of tanks also.

Table 4.2   Effect of cost ratio R_2 on optimal radius

$$\beta = 1.0$$

Cost ratio R_1	Capacity (m^3)	Optimal radius (mm)		
		$R_2 = 40$	$R_2 = 100$	$R_2 = 160$
600	750	7149	7191	7225
	1000	8097	8179	8207
	1500	9737	9798	9831
1200	500	5565	5670	5745
	750	6603	6711	6784
	1500	9055	9086	9128
1800	250	3984	4053	4162
	500	5273	5380	5470
	1250	7891	7946	8016

Table 4.3   Effect of degree of fixity β on optimal radius

$$R_2 = 100$$

Cost ratio R_1	Capacity (m^3)	Optimal radius (mm)					
		$\beta=1.0$	$\beta=0.8$	$\beta=0.6$	$\beta=0.4$	$\beta=0.2$	$\beta=0.0$
600	750	7191	7185	7183	7203	7225	7243
	1000	8179	8165	8134	8119	8155	8186
	1500	9798	9720	9694	9692	9688	9730
1200	500	5670	5660	5630	5634	5639	5646
	750	6711	6700	6687	6677	6685	6691
	1500	9086	9077	9027	9014	9023	9043
1800	250	4053	4049	4051	4051	4055	4057
	500	5380	5365	5353	5350	5356	5360
	1250	7946	7933	7887	7976	7872	7894
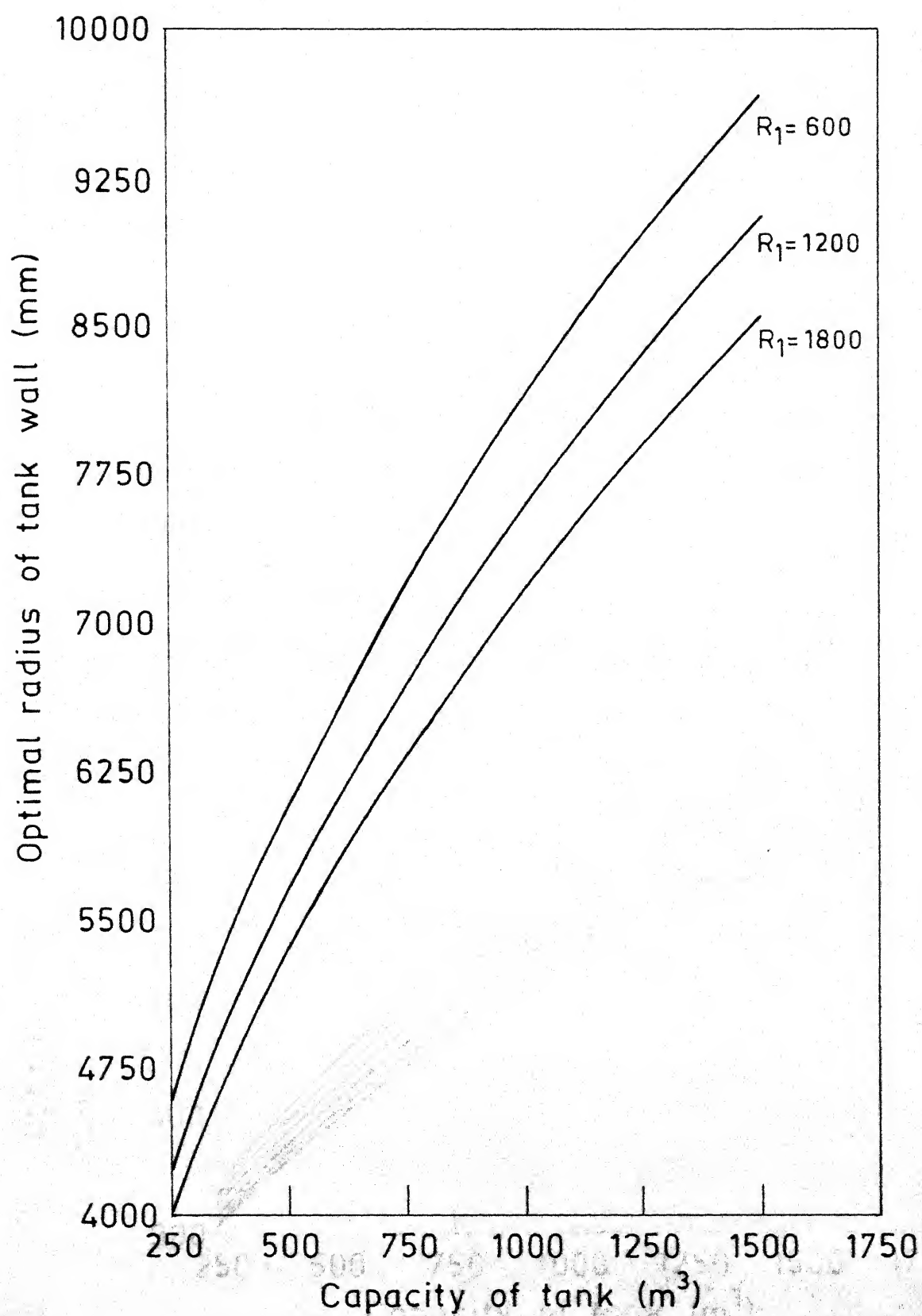
Fig. 4.3 - Design chart for optimal radius of tank wall.

Fig. 4.4 - Design chart for optimal areas of hoop and vertical reirforcements.
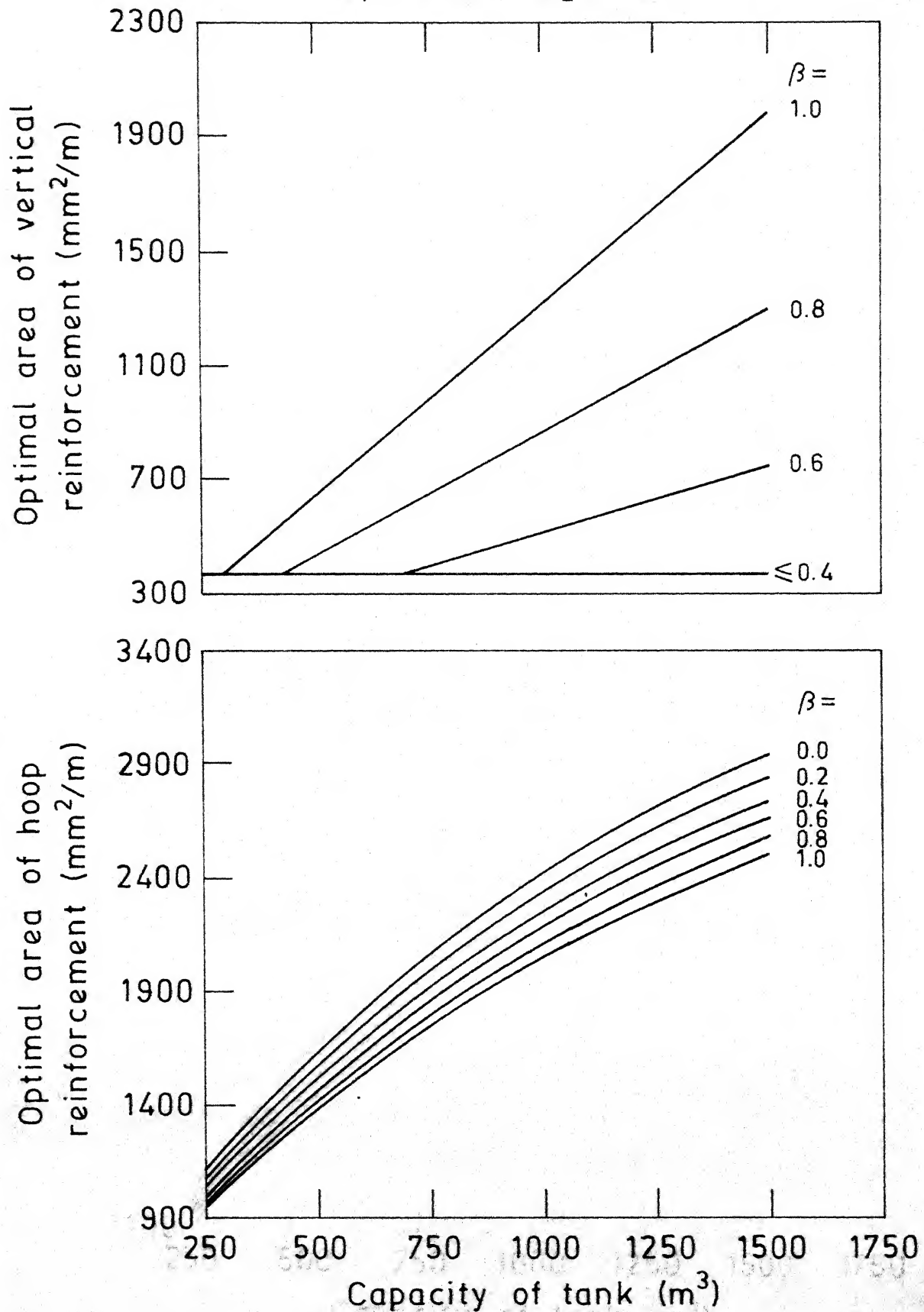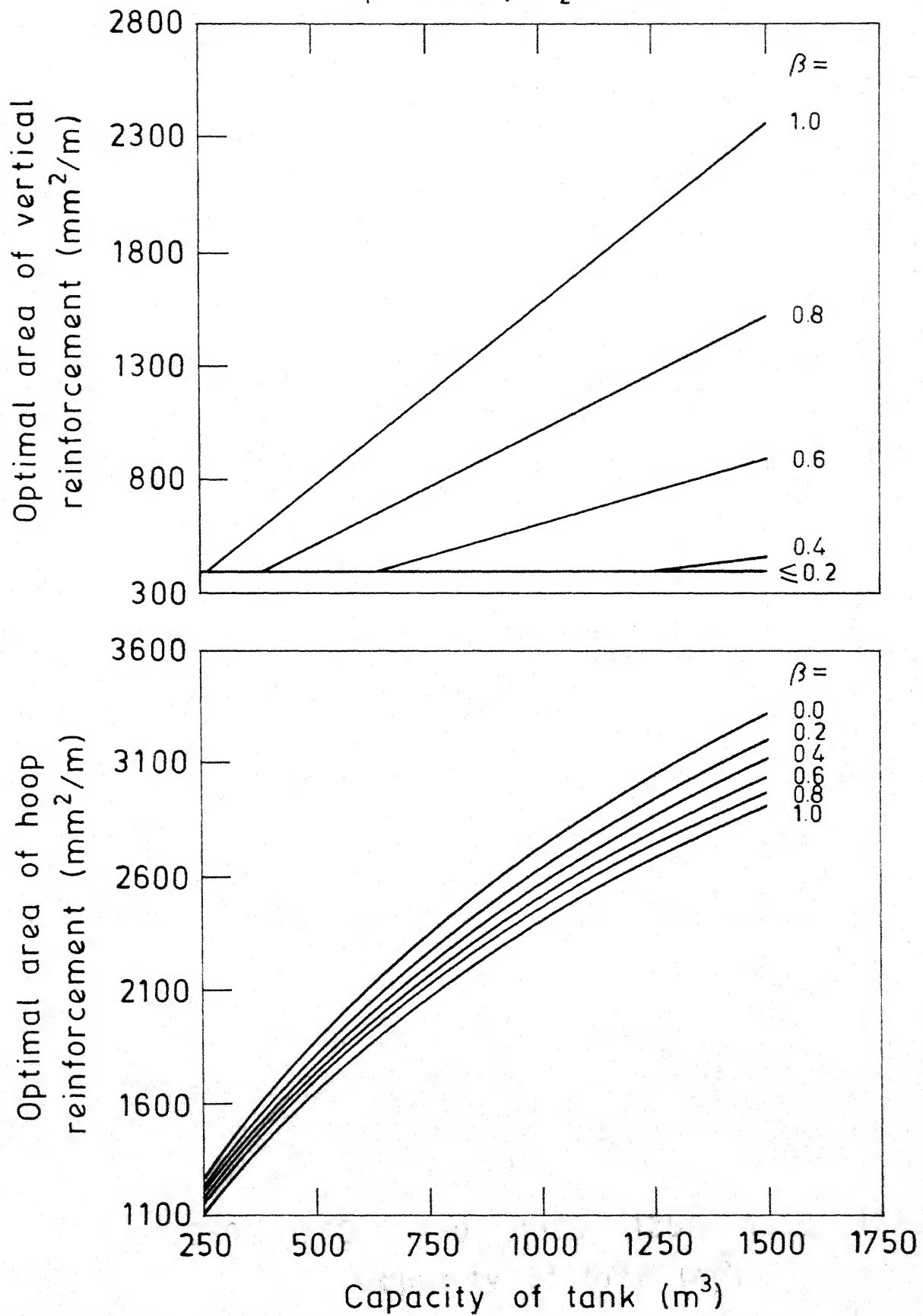
Fig. 4.5 – Design chart for optimal areas of hoop and vertical reinforcements.
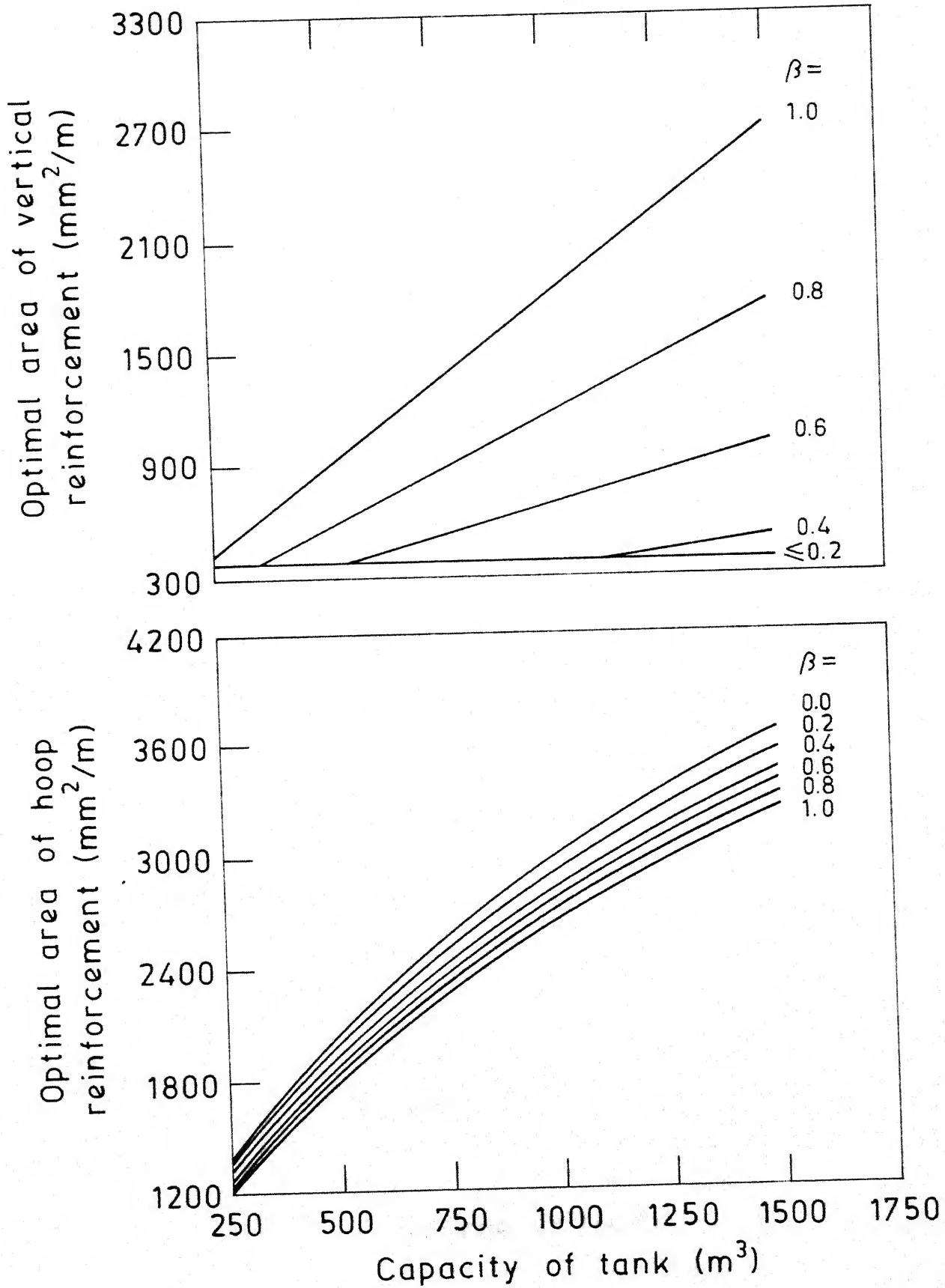
Fig. 4.6 - Design chart for optimal areas of hoop and vertical reinforcements.
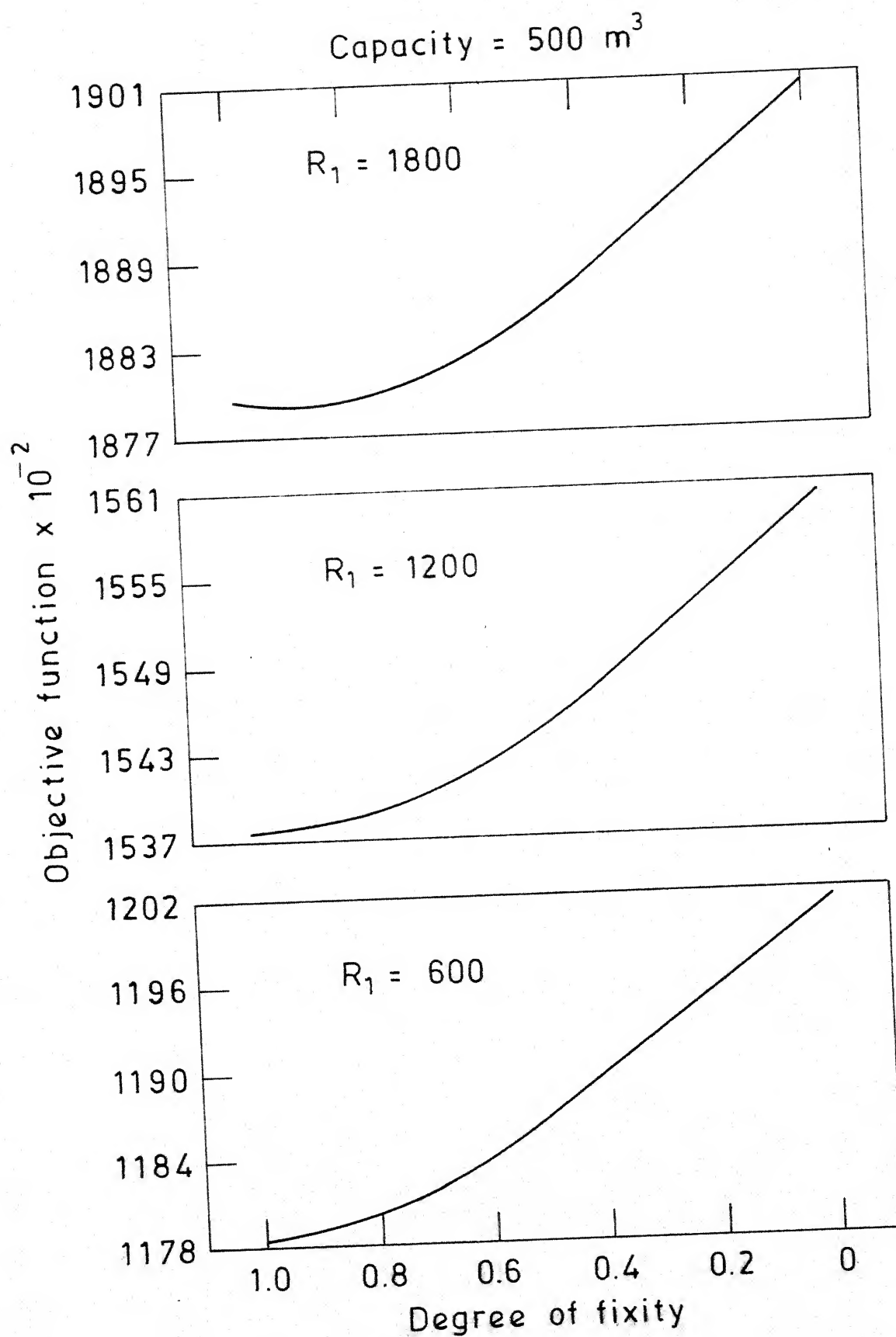
Fig. 4.7 - Effect of degree of fixity on objective function.
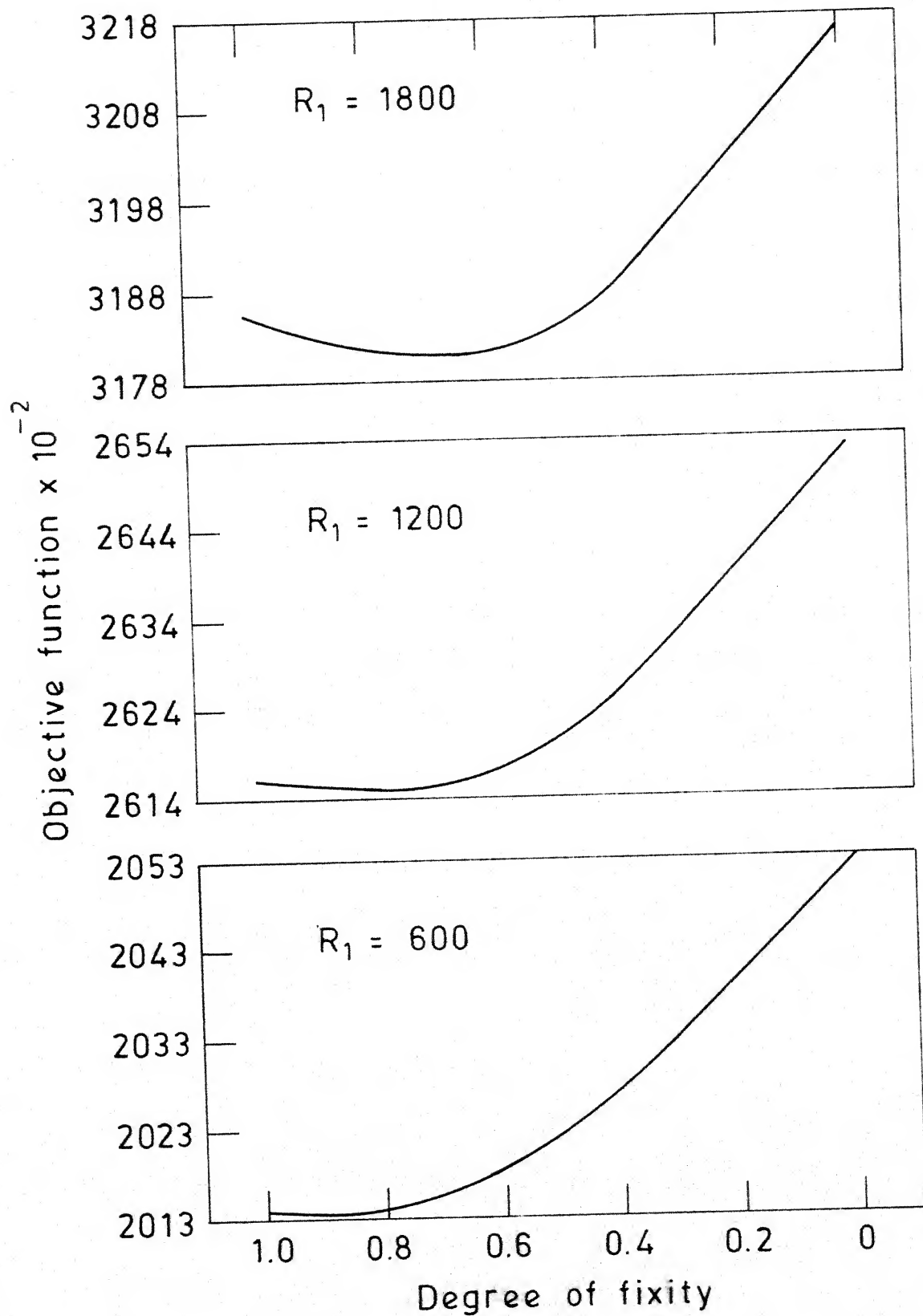
Fig. 4.8 - Effect of degree of fixity on objective function.
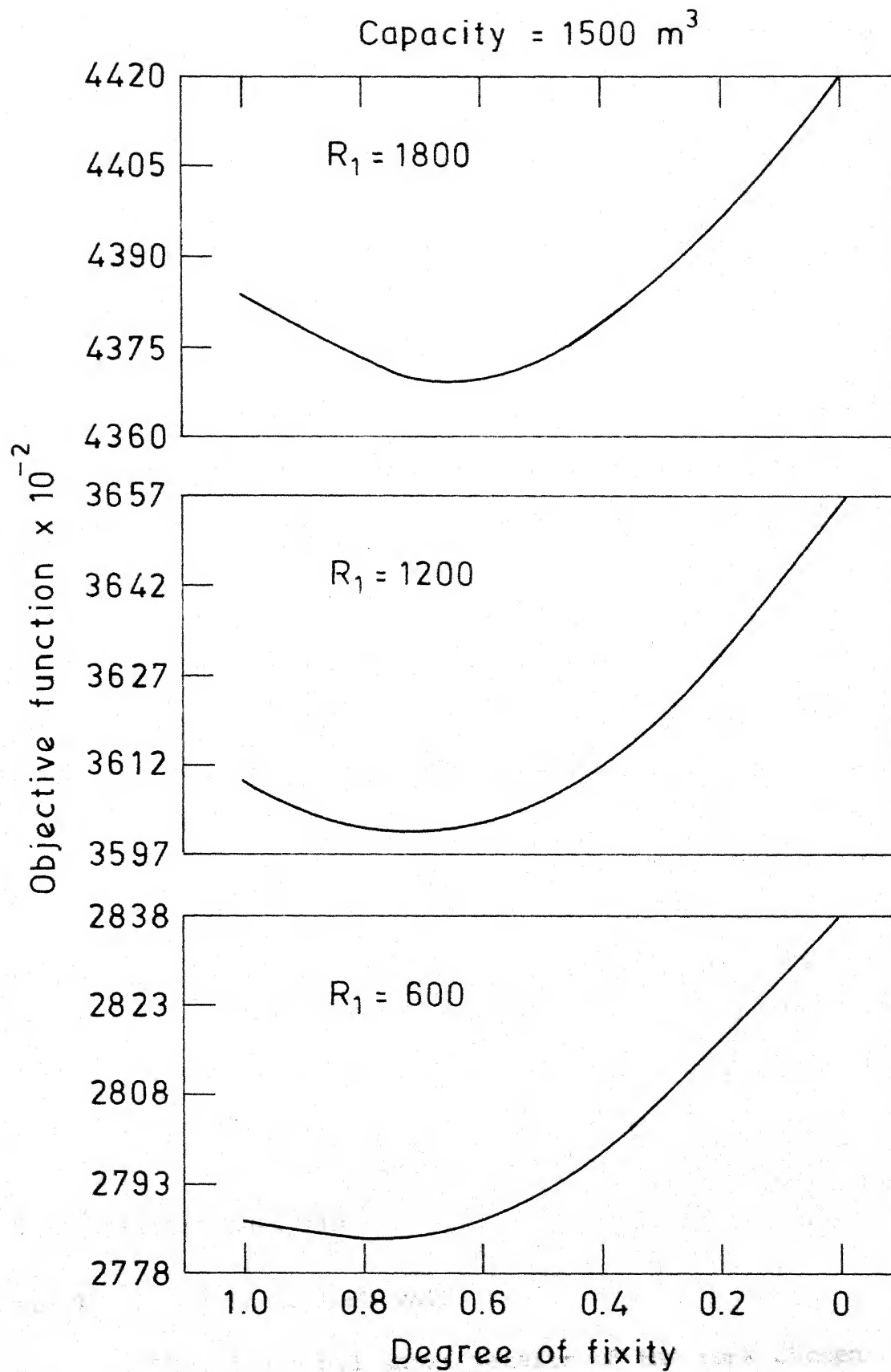
Fig. 4.9 - Effect of degree of fixity on objective function.

CHAPTER 5

OPTIMAL DESIGN OF OVERHEAD WATER TANKS

5.1 Design Considerations

Overhead water tanks are constructed either for water supply or for other purposes which require water under pressure. Overhead cylindrical tanks are designed with many types of bottoms. Flat floors with or without a system of beams, combination of sloping and flat bottoms, combination of sloping and curved bottoms are some of the alternatives. In many countries, complicated forms of the bottom, such as that of Intze, are so expensive that structures of simpler shapes, although requiring more concrete and steel, are cheaper and sounder (Gray and Manning, 1973). In this chapter, cylindrical tanks with a domed roof and a flat bottom are considered. The floor slab along with the tank is carried either by a wall or a tower. A tower diameter of about 0.75 to 0.8 times that of the tank wall has generally resulted in economical designs.

5.2 Elastic Analysis

5.2.1 Analysis of tank wall

The Figure 5.1 shows details of the tank chosen for design, in which R_t represents radius of the tower and h_f, thickness of the floor slab. Analysis of joint 1 is
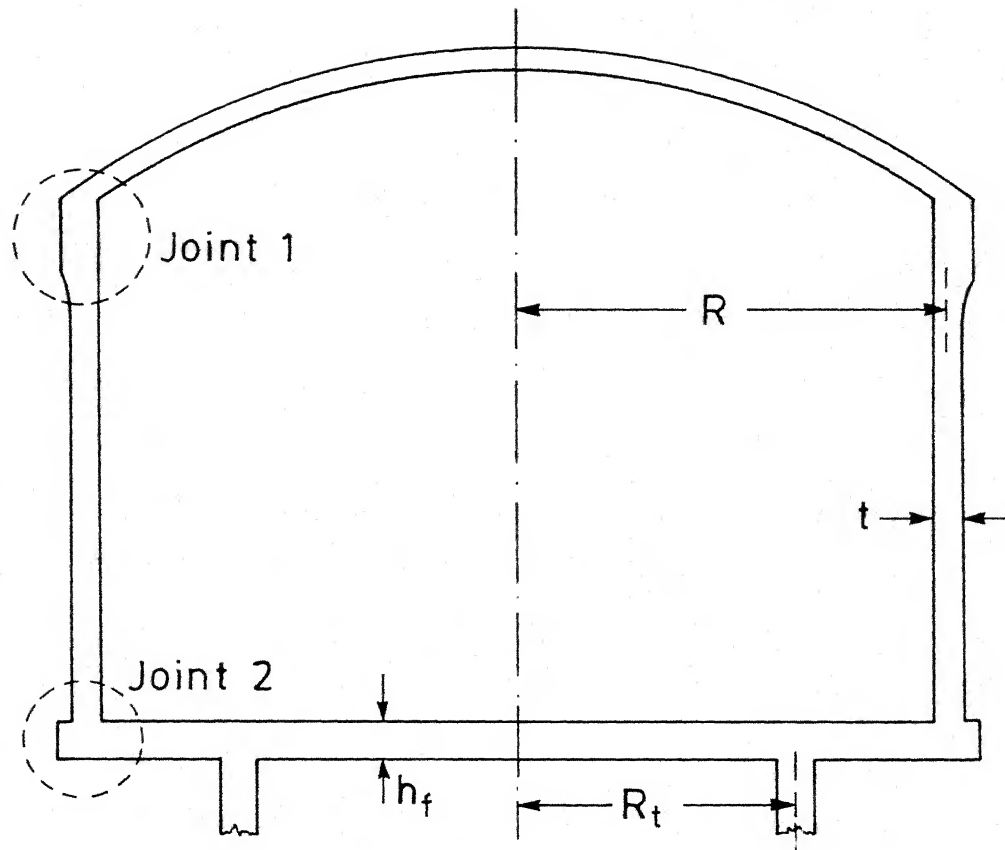
Fig. 5.1 - Details of the overhead tank

$$\frac{qR^2}{8D_p}$$

$$\frac{P}{4\pi D_p}\left(\frac{R^2-R_t^2}{R}\right)$$

$$\frac{MR}{D_p}$$
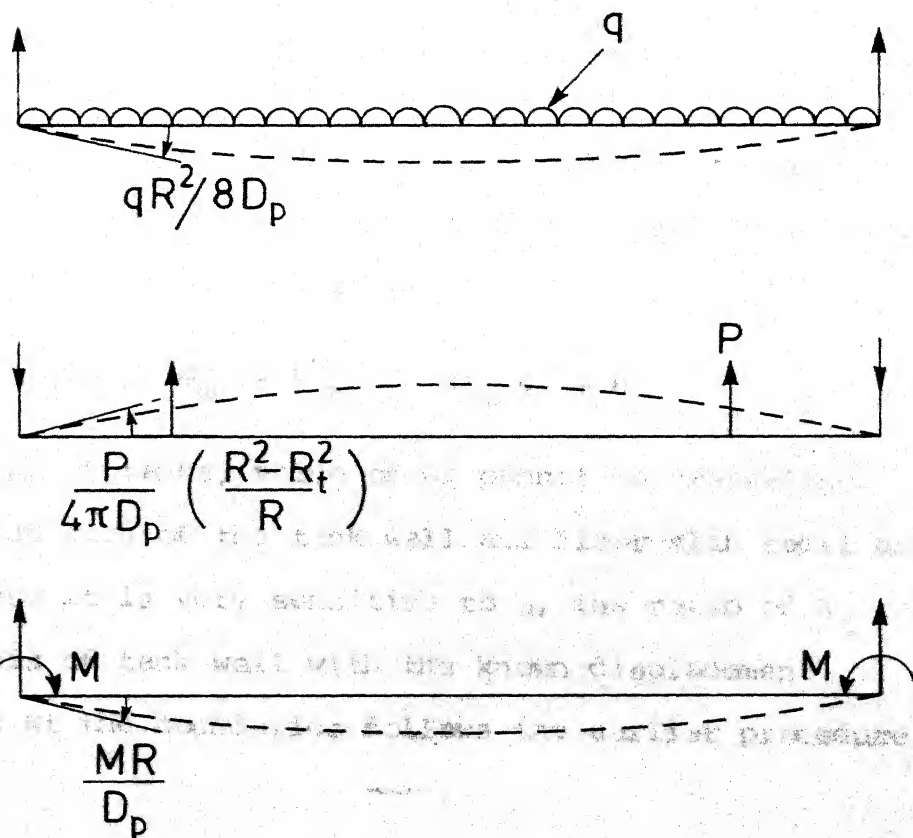
Fig. 5.2 - Displacements in a simply supported plate.

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exactly similar to the one carried out in Section 4.2. At
joint 2, deformation of the floor slab has to be taken into
account in this case. The nature of displacement of a simply
supported circular plate subjected to three different types
of load are shown in Figure 5.2. The moment required to
cause unit rotation of the plate edge can be obtained from
the third loading case and is given by

$$k_{mm}^p = D_p/R , \qquad\qquad ..(5.1)$$

where D_p is the flexural rigidity of the plate $= Eh_f^3/12$.
Clockwise rotation of the plate due to the combined action
of a distributed load q and a line load of total magnitude P
is given by

$$\Theta_p = \frac{qR^2}{8D_p} - \frac{P}{4\pi D_p} \left(\frac{R^2 - R_t^2}{R}\right) . \qquad\qquad ..(5.2)$$

Value of Θ_2, the final anticlockwise rotation of joint 2,
can be obtained from consideration of the joint equilibrium
equation which can be written as

$$k_{mm}^w(\Theta_2 + \Theta_w) - k_{mh}^w x_w + k_{mm}^p(\Theta_2 + \Theta_p) = 0 . \qquad ..(5.3)$$

In this type of tanks, value of Θ_2 cannot be preassigned
but is a function of the tank wall and floor slab radii and
thicknesses; it is very sensitive to α, the ratio of R_t to
R. Analysis of tank wall with the known displacement
conditions at the boundaries follows the earlier procedure.

5.2.2   Analysis of floor slab

The floor slab is treated as a circular plate subjected to a uniform load of intensity q, edge moment M, edge line load from the tank dome and wall, and supported by a wall or tower of radius R_t by a line load of total magnitude P. The deflection in such a plate is given by

$$w = \frac{q}{64D_p} (R_t^4 - r^4) - \frac{3qR^2}{32D_p} (R_t^2 - r^2) + \frac{M}{2D_p} (R_t^2 - r^2)$$

$$- \frac{P}{8\pi D_p} \left[(R_t^2 - r^2)(1 + \frac{R^2 - R_t^2}{2R^2}) + (R_t^2 + r^2) \log \frac{r}{R} \right.$$

$$\left. - 2R_t^2 \log \frac{R_t}{R} \right], \quad \text{for } R_t \leqslant r \leqslant R, \quad \text{..(5.4)}$$

and

$$w = \frac{q}{64D_p} (R_t^4 - r^4) - \frac{3qR^2}{32D_p} (R_t^2 - r^2) + \frac{M}{2D_p} (R_t^2 - r^2)$$

$$- \frac{P}{8\pi D_p} \left[(R_t^2 - r^2) \frac{(R^2 - R_t^2)}{2R^2} - (\log \frac{R_t}{R})(R_t^2 - r^2) \right],$$

$$\text{for } 0 \leqslant r \leqslant R_t . \quad \text{..(5.5)}$$

Expressions for the radial moment M_r and tangential moment M_t are obtained from these equations for deflection. The value of these moments at different locations of the plate are given by the following expressions:

$$M_r \big|_{r=0} = M_t \big|_{r=0} = \frac{3qR^2}{16} + \frac{P}{8\pi} (2\log \frac{R_t}{R} + \frac{R_t^2}{R^2} - 1) + M,$$

$$\text{..(5.6)}$$

$$M_r \Big|_{r = R_t} = \frac{3q(R^2 - R_t^2)}{16} + \frac{P}{8\pi} \left(2\log \frac{R_t}{R} + \frac{R_t^2}{R^2} - 1\right) + M \ , $$

$$..(5.7)$$

$$M_t \Big|_{r = R_t} = \frac{q(3R^2 - R_t^2)}{16} + \frac{P}{8\pi} \left(2\log \frac{R_t}{R} + \frac{R_t^2}{R^2} - 1\right) + M \ , $$

$$..(5.8)$$

$$M_r \Big|_{r = R} = M = -k\frac{P}{mm}(\Theta_z + \Theta_p) \ , \qquad\qquad ..(5.9)$$

and

$$M_t \Big|_{r = R} = \frac{qR^2}{8} + \frac{P}{4\pi} \left(\frac{R_t^2}{R^2} - 1\right) + M \ . \qquad\qquad ..(5.10)$$

Distribution of these moments is shown in Figure 5.3. A suitable pattern of reinforcement to resist these moments, keeping in view the relative economy as well as feasibility, is arrived at after considering some of the alternative patterns, and is shown in Figure 5.4. Reinforcement required near the bottom surface is comparatively less and after optimization it is found to correspond to the minimum percentage (0.3%). Therefore a uniform mesh has been adopted. On the other hand, radial moment over the support needs more reinforcement. By providing the reinforcement in the radial pattern, since the spacing of reinforcement will decrease with a decrease in the radial distance, reinforcement will be more effective at the support than at the periphery; curtailment of reinforcement will also be simpler. Near the top surface, nominal reinforcement of 0.12% has been provided in the

Variation of M_r Variation of M_t
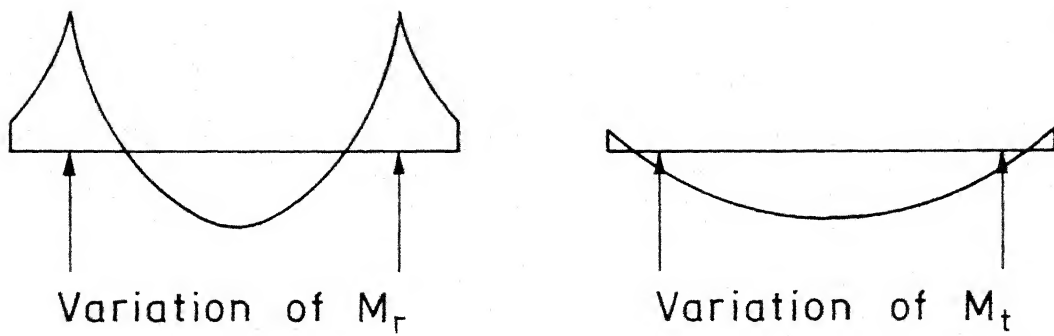
Fig. 5.3 - Distribution of moments in the floor slab.

Reinforcement near
top surface
0.5 a_{p1} from 0.5 R to 0.6 R
a_{p1} from 0.6 R to R

Reinforcement near
bottom surface
uniform mesh a_{p2} in
both directions
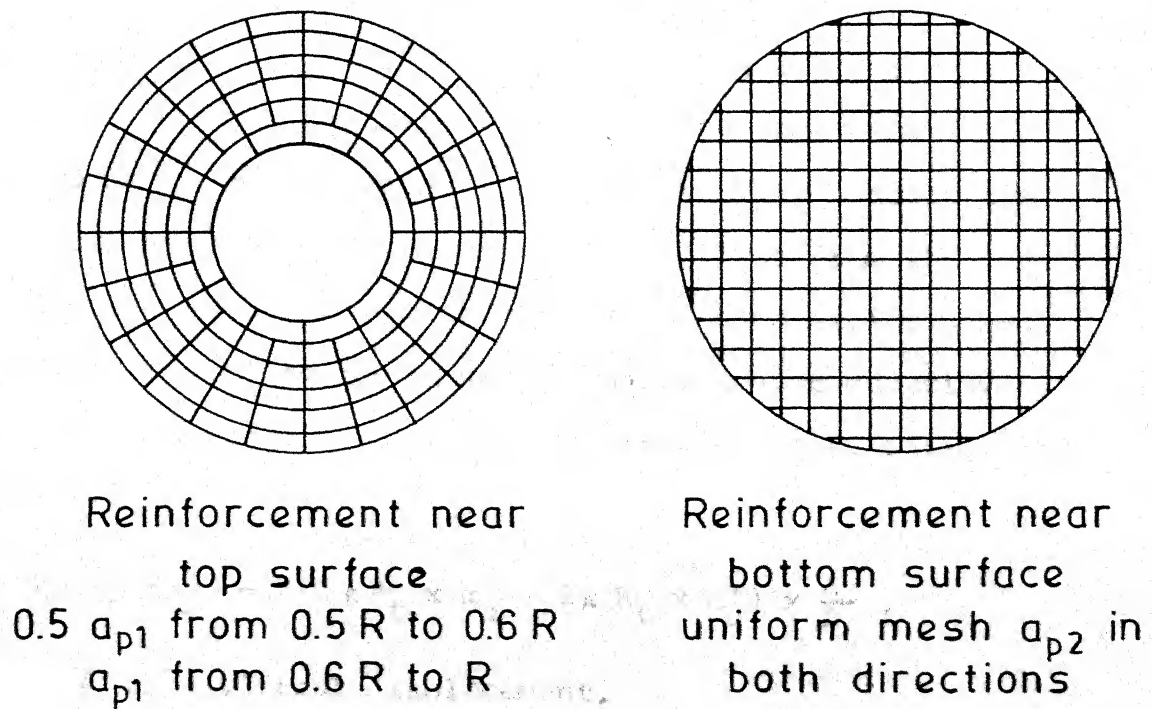
Fig. 5.4 - Details of reinforcement arrangement in the
floor slab.

circumferential direction up to 0.5 R from the periphery to take care of negative circumferential moments and for proper distribution of forces.

5.3 Limit Analysis

5.3.1 Analysis of tank wall

Mode of failure of the tank wall is again seen to correspond to the partial collapse discussed in Section 3.3.6.

5.3.2 Analysis of floor slab

There are two possible collapse mechanisms for the floor slab which are shown in Figure 5.5. The first mode corresponds to a partial collapse of the inner portion of the slab due to formation of a negative hinge circle over the support and positive radial yield lines from the centre to the support. If m_u corresponds to the positive moment capacity and m_u' to the negative radial moment capacity at the support, then the load q_u required to cause failure by this mode is given by

$$q_u \times \pi R_t^2 \times \frac{\Delta}{3} = (2\pi R_t \times m_u + 2\pi R_t \times m_u') \times \frac{\Delta}{R_t} , \qquad ..(5.11)$$

where Δ is a virtual displacement.

Hence,
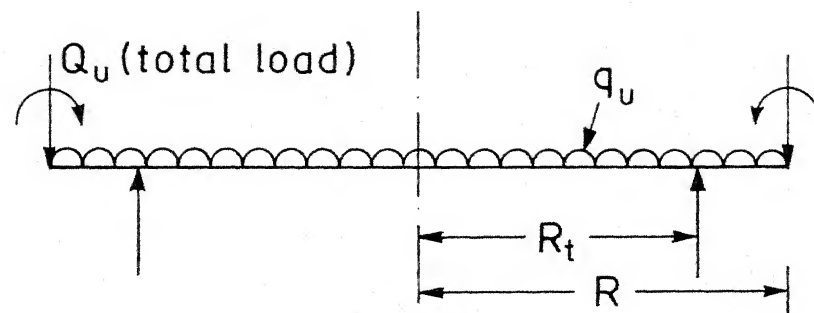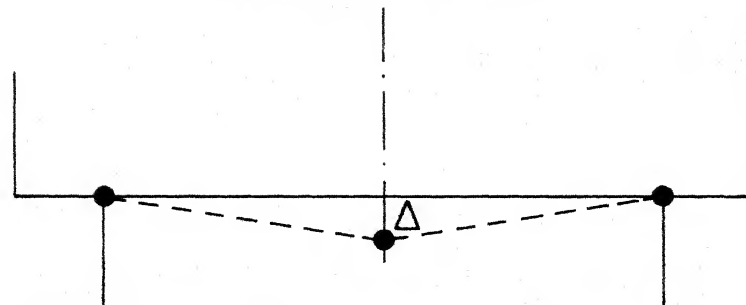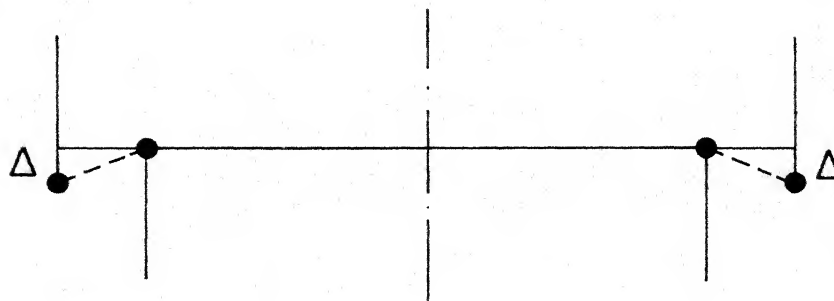
$$q_u = \frac{6(m_u + m_u')}{R_t^2} . \qquad ..(5.12)$$

Fig. 5.5 - Collapse modes for the floor slab.

The second mode corresponds to the partial collapse of the slab between the support and periphery. This is caused by formation of a negative hinge circle at the support and a positive hinge circle at the periphery connected by radial yield lines. If the moment capacities to resist the formation of these yield lines are m_u', m_{up} and m_u'' respectively, then equating the external work to the internal work due to a virtual displacement Δ, the following relationship is obtained:

$$Q_u \times \Delta + q_u \times \pi (R^2 - R_t^2) \times \frac{\Delta}{3} =$$

$$\frac{[2 \pi R_t \times m_u' + 2 \pi R \times m_{up} + 2 \pi (R - R_t) \times m_u''] \times \Delta}{(R - R_t)} , \qquad ..(5.13)$$

where Q_u is the factored value of the line load transmitted by the tank wall.

Value of the load q_u required to cause collapse is the lesser of the two values given by Eqs. (5.12) and (5.13).

5.4 Optimization

5.4.1 Design variables

In addition to the design variables chosen earlier for the optimal design of the tank wall, namely, R, t, a_{h2} and a_{v2}, additional design variables are required for the optimal design of the floor slab. These are, the thickness of the floor slab h_f, ratio of radii α, area of reinforcement

provided near the top surface at the periphery a_{p1} and area of reinforcement provided near the bottom surface a_{p2}.

5.4.2   Objective function

The objective function considered is the combined costs of concrete, reinforcement, and formwork required for the dome with ring beam, tank wall and floor slab. The unit cost of formwork for the floor slab is taken as 50% of that for curved surfaces. Writing the objective function as

$$F = F_f + F_w + F_b + F_d , \qquad\qquad ..(5.14)$$

where F_f is the cost of floor slab, F_w cost of wall, F_b cost of ring beam and F_d cost of dome. Value of $(F_b + F_d)$ can be obtained from Eq. (4.16). Expressions for finding F_w and F_f are as follows:

$$F_w = R_1 \times 2\pi Rtl + 2\pi R \gamma_s \times [(a_{h1}H_1 + a_{h2}H_2 + a_{h3}H_3$$
$$+ a_{v1}l_1 + a_{v2}l_2)] + R_2 \times 4\pi Rl , \qquad\qquad ..(5.15)$$

$$F_f = R_1 \times \pi R^2 h_f + \pi R^2 \gamma_s \times (0.9\ a_{p1} + 2a_{p2}$$
$$+ 0.0009\ h_f) + \frac{R_2}{2} \times \pi R^2 . \qquad\qquad ..(5.16)$$

5.4.3   The constraints

In addition to the 12 constraints discussed in Section 3.4.3, additional constraints, for the satisfactory functioning of the floor slab at both service and ultimate loads, must be prescribed. The values of thickness h_f and

areas of reinforcement a_{p1} and a_{p2} must be sufficient enough to ensure that the crack widths, w_{cr3} due to negative radial moment at $r = R_t$ and w_{cr4} due to positive moment at the centre, are less than 0.2 mm. Thus the 13th and 14th constraints may be stated as

$$\frac{w_{cr3}}{0.2} - 1 \leqslant 0 , \qquad\qquad ..(5.17)$$

and

$$\frac{w_{cr4}}{0.2} - 1 \leqslant 0 . \qquad\qquad ..(5.18)$$

The next two constraints deal with the minimum value a_{p1} and a_{p2} can take. Thus

$$\frac{a_{min}}{a_{p1}} - 1 \leqslant 0 , \qquad\qquad ..(5.19)$$

and

$$\frac{a_{min}}{a_{p2}} - 1 \leqslant 0 . \qquad\qquad ..(5.20)$$

The ultimate load capacity q_u', obtained after deducting the dead load effects from q_u as predicted from the yield line analysis, should be at least equal to the factored water pressure. The 17th constraint can therefore be written as

$$\frac{1.6 \, \gamma_w l}{q_u'} - 1 \leqslant 0 . \qquad\qquad ..(5.21)$$

5.5   Example

5.5.1   Data

Optimal limit state designs of an elevated water tank of the type under consideration, with a capacity of 200 m^3, is obtained for three values of cost ratio R_1. The results are compared with that of an indirect design employing working stress method and also with that of a textbook solution (Gray and Manning, 1973) obtained for a similar tank, but with a flat roof and designed for a capacity of 204 m^3 (45,000 gallons).

5.5.2   Working stress design

Quite a few trials were required in order to arrive at a suitable value of α, t and h_f so that the materials are stressed to their permissible values at critical sections. Choosing R = 4 m, α = 0.79, h_f = 350 mm and t = 150 mm, moments, in kNm/m, in the floor slab due to service loads are as follows:

radial moment at periphery = - 7.685,

radial moment at support = - 52.387,

moment at the centre = 39.218,

circumferential moment at periphery = 1.553, and

circumferential moment at support = 8.683.

In order to control crack widths, maximum flexural tensile stress in concrete, based on an uncracked section,

should not exceed 1.84 MPa and maximum tensile stress in steel, based on a cracked section, should not exceed 130 MPa. Using a value of a_{p2} = 1120 mm^2/m, these values are found to be 1.75 MPa and 129 MPa respectively, at the centre. Similarly, using a value of a_{p1} = 3715 mm^2/m, the relevant stresses are 1.83 MPa and 44.5 MPa respectively, at the support.

Although the stress in reinforcement at the support is low, the area of reinforcement cannot be reduced as it will lead to an increase in the flexural tensile stress in concrete. If an attempt is made to avoid this by increasing the thickness of slab, value of a_{p2} which almost corresponds to the minimum will also increase, with the result that the materials will be under-stressed at the centre.

Forces in the tank wall controlling its design are a moment of 7.685 kNm/m at the base and a maximum hoop tension of 106.5 kN/m. With a wall thickness of 150 mm, required areas of various reinforcements, so that the relevant stresses are not exceeded, are a_{v1} = 450 mm^2/m (minimum), a_{v2} = 1500 mm^2/m (to control flexural tensile stress in concrete), a_{h1} = 590 mm^2/m (designed for class A exposure), a_{h2} = 820 mm^2/m and a_{h3} = 450 mm^2/m (minimum). Values of H_1, H_2 and H_3 are 1.335 m, 2.2 m and 0.6 m respectively.

5.5.3   Textbook solution

Two solutions are given in the textbook, one which satisfies provisions of CP 2007 (1970) and the other corresponding to that of a competetive design. Comparison will be

restricted to the one that satisfies the code requirements as the other design has no basis. But, it is obvious that water tanks built on the basis of such competetive designs have stood the test of time and hence the provisions of CP 2007 and similar codes based on working stress method lead to excessively safe and uneconomical designs.

The tank wall is designed to have a radius of 3.875 m and a thickness of 150 mm. The floor slab is 350 mm thick and supported on a wall with $\alpha = 0.8$. The reinforcements provided are $a_{h2} = 1800$ mm^2/m, $a_{v2} = 735$ mm^2/m, $a_{p1} = 1765$ mm^2/m and $a_{p2} = 2390$ mm^2/m.

The roof slab requires a thickness of 275 mm and a reinforcement of 1810 mm^2/m both ways. Nothing has been mentioned about curtailment of reinforcement. For comparisons to be meaningful, the objective function for the textbook solution is computed on the basis of a domed roof and the same pattern of curtailment of reinforcement as followed for the other designs.

5.5.4 Comparison

A comparison of different designs of this overhead tank is given in Table 5.1. Since the values of design variables for the indirect design and textbook solution do not vary with change in the value of R_1, they have been listed only once. Indirect design and textbook design cost respectively an average of about 28% and 37% more than the

Table 5.1 Comparison of different designs of an overhead tank

$R_2 = 100$

Cost ratio R_1	Nomenclature	R (mm)	t (mm)	a_{v2} (mm²/m)	a_{h2} (mm²/m)	α	h_f (mm)	a_{p1} (mm²/m)	a_{p2} (mm²/m)	Objective function
600	Indirect design	4000	150	1500	820	0.790	350	3715	1120	84599
	Textbook design	3875	150	735	1800	0.800	350	1765	2390	92322
	Optimal design	3730	125	375	980	0.786	250	815	.755	66466
1200	Indirect design									108074
	Textbook design									115251
	Optimal design	3600	125	375	1065	0.782	235	950	710	83647
1800	Indirect design									131548
	Textbook design									138180
	Optimal design	3560	125	375	1095	0.780	230	1010	720	100760
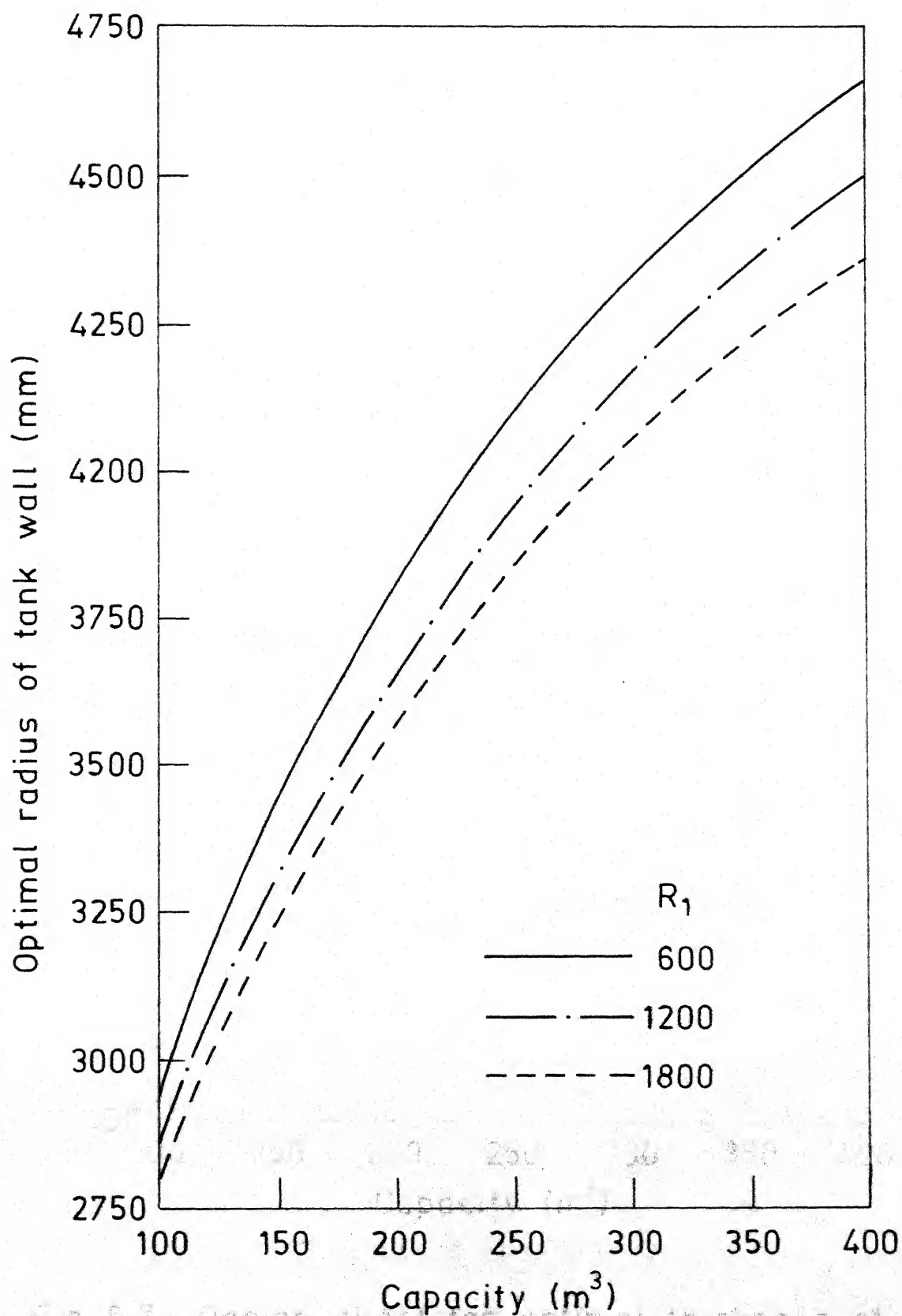
Fig. 5.6 - Design chart for optimal radius of tank wall.
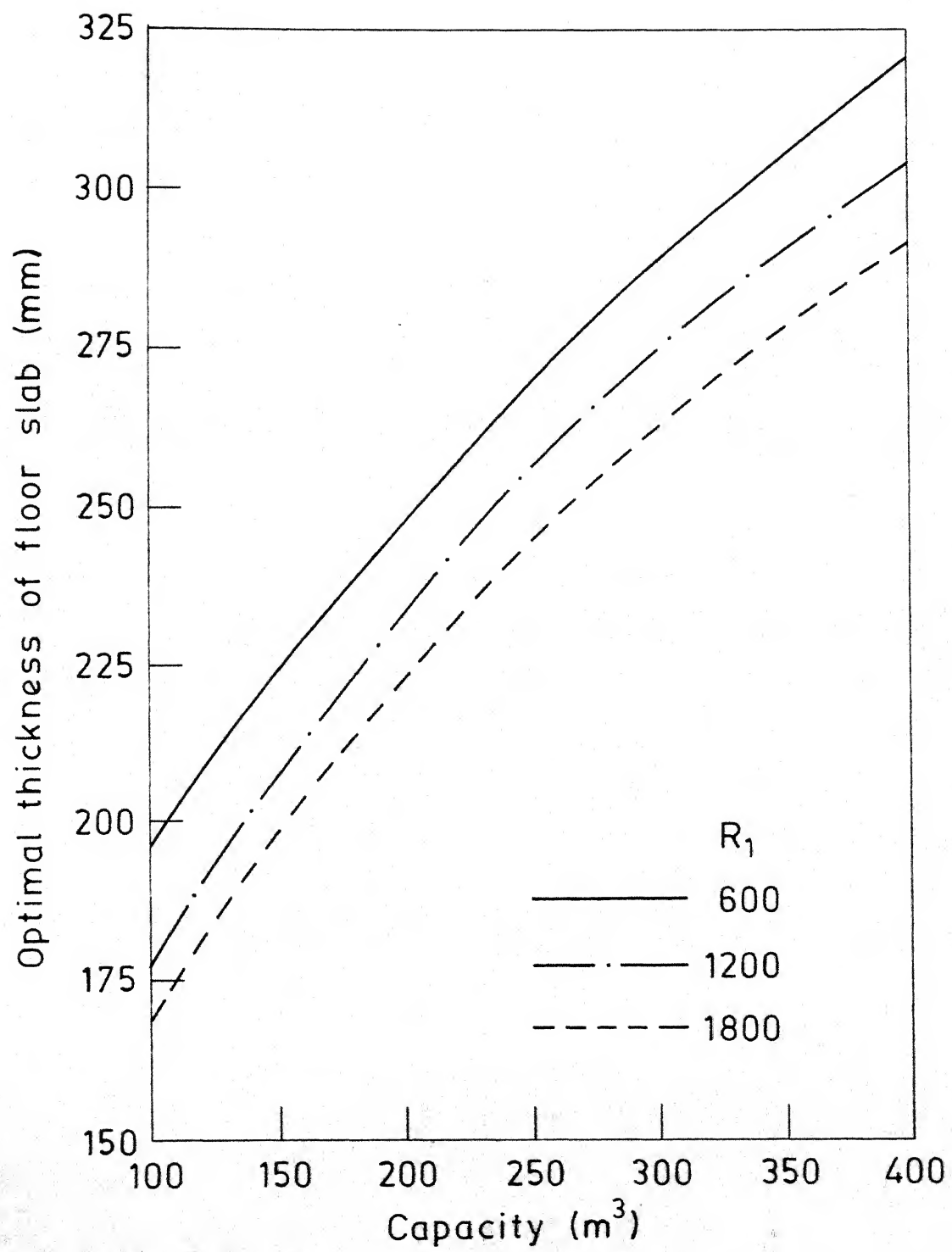
Fig. 5.7 - Design chart for optimal thickness of floor slab.

Table 5.2   Optimal values of design variables and objective function

$R_1 = 600;$ $R_2 = 100$

Capacity (m³)	R (mm)	α	t (mm)	t_p (mm)	a_{v2} (mm²/m)	a_{h2} (mm²/m)	a_{p1} (mm²/m)	a_{p2} (mm²/m)	Objective function
100	2932	0.802	125	196	375	608	620	588	38743
150	3452	0.786	125	215	375	780	813	648	52578
200	3730	0.786	125	250	375	978	815	753	66466
250	4106	0.775	125	268	375	1120	937	805	79734
300	4317	0.769	125	285	375	1311	1007	857	92768
350	4491	0.763	125	301	375	1509	1107	903	105981
400	4718	0.759	125	325	375	1647	1089	986	119258

Table 5.3   Optimal values of design variables and · objective function

$R_1 = 1200;$ $R_2 = 100$

Capacity (m^3)	R (mm)	α	t (mm)	t_p (mm)	a_{v2} (mm^2/m)	a_{h2} (mm^2/m)	a_{p1} (mm^2/m)	a_{p2} (mm^2/m)	Objective function
100	2848	0.789	125	176	375	674	790	531	48691
150	3310	0.785	125	207	375	854	900	639	66505
200	3598	0.782	125	234	375	1063	952	711	83647
250	3925	0.770	125	257	375	1243	997	786	100636
300	4110	0.764	125	266	375	1462	1177	807	116615
350	4323	0.763	125	289	375	1617	1217	867	133074
400	4585	0.758	125	299	375	1744	1327	963	148675

Table 5.4   Optimal values of design variables and objective function

$R_1 = 1800$; $R_2 = 100$

Capacity (m^3)	R (mm)	α	t (mm)	t_p (mm)	a_{v2} (mm^2/m)	a_{h2} (mm^2/m)	a_{p1} (mm^2/m)	a_{p2} (mm^2/m)	Objective function
100	2809	0.780	125	170	375	720	871	536	58643
150	3246	0.774	125	195	375	927	1046	664	80175
200	3561	0.780	125	230	375	1092	1010	720	100760
250	3850	0.769	125	247	375	1292	1118	763	120590
300	4048	0.762	125	262	375	1512	1243	810	140025
350	4104	0.761	125	273	375	1780	1383	905	159288
400	4333	0.757	125	288	375	1930	1455	924	178135

R_1 = 600. The value of α goes on decreasing gradually with an increase in the value of both R_1 and the capacity of tank; for a tank of capacity 400 m^3 with R_1 = 1800, α has a minimum value of 0.757. This is because of the fact, that the edge moment is very sensitive to the value of α and decreases with a decrease in the value of α. For any capacity and value of R_1, α gets adjusted to a value which results in the maximum value of edge moment which can be resisted by a tank wall with minimum thickness and minimum value of a_{v2} without violating the crack width constraint. Thus, the variables t and a_{v2} have assumed the minimum values in all the cases.

Thickness of floor slab varies from 170 mm to 325 mm for different capacities and cost ratios considered. For a given capacity, it decreases with an increase in the value of R_1; but, the variation is not large. While a tank of capacity 200 m^3 when designed by the working stress method required a floor slab of 350 mm thickness, that with a capacity of 400 m^3, can be designed by the limit state method, with a 300 mm thick floor slab. Substantial reduction in the thickness and reinforcement required for the floor slab when designed by the limit state method will enable choosing this type of tank with advantage rather than those with complex forms of bottom, not only for small capacities but for medium capacities also.

Constraints which are critically satisfied are those regarding the following:

(1)   crack width in the tank wall at the bottom,

(2)   minimum thickness of tank wall,

(3)   minimum values of vertical reinforcement,

(4)   maximum stress in the hoop reinforcement,

(5)   crack widths in the floor slab, both at the support and at the centre,

(6)   minimum area of reinforcement in the central region of the floor slab, and

(7)   strength of the floor slab, except in the case of smaller capacities.

CHAPTER 6

COMPUTATIONAL ASPECTS

6.1   General

Mathematical programming techniques, with the exception of classical methods, involve lot of numerical computations. It was only after the advent of high speed digital computers that the numerical methods came into prominence and considerable progress achieved during the 1950s and 1960s. In this chapter, computational details of the optimization techniques used in this study, organization of the computer programs, and related aspects are presented. Details of the Newton-Raphson method, which has been employed for the solution of both transcendental equations and a system of nonlinear simultaneous equations, have also been given for the sake of completeness. All the computations have been carried out on a DEC-1090 system at the Indian Institute of Technology, Kanpur.

6.2   Algorithms for the Optimization Methods Employed

The constrained nonlinear programming problems of the current study have been converted into unconstrained nonlinear programming problems through the use of interior penalty functions as outlined in Section 1.4.4; Davidon-Fletcher-Powell method (DFP method) to find the search

direction $\overline{S}$ and cubic interpolation method to find the appropriate step length α, are then employed to find the minimum of the modified objective function. The steps involved in the solution of the constrained optimization problem may be summarized as follows:

(1)   The solution is commenced from a feasible point $\overline{X}_1$, which satisfies all the constraints with strict inequality sign. A suitable value is chosen for the penalty parameter r and the counter k set equal to 1.

(2)   The modified objective function $\emptyset(\overline{X}, r_k)$ is minimized to get $\overline{X}_k^*$ using DFP algorithm and cubic interpolation method as explained in steps (6) through (11).

(3) $\overline{X}_k^*$ is tested for optimality. If it is optimal, the process is terminated.

(4)   Otherwise, the penalty parameter is modified as $r_{k+1} = cr_k$, where c is less than 1.

(5)   The new value of k is set equal to k+1, the new starting point $\overline{X}_1$ set equal to $\overline{X}_k^*$ and the next minimization cycle commenced from step 2.

The DFP algorithm and the cubic interpolation method can be summarized as follows:

(6)   The algorithm starts with the initial point $\overline{X}_1$ and a m x m positive definite symmetric matrix $[H]_1$, where m is the number of design variables. The iteration number is set equal to 1.

(7)   The gradient of the modified objective function, $\nabla\phi_i$, at the point $\overline{X}_i$ is computed from which the search direction $\overline{S}_i$ is found as

$$\overline{S}_i = - [H]_i \, \nabla\phi_i \, . \qquad ..(6.1)$$

(8)   The minimizing step length α_i^* in the direction $\overline{S}_i$ is found, using the cubic interpolation method, in four stages. First, the search direction $\overline{S}_i$ is normalized so that a step size $\alpha = 1$ is acceptable. Then, the directional derivatives of the function ϕ are used to establish bounds on α^* as the slope has to change from a negative value to a positive value, if the minimum has been bracketed. In the third stage, an approximate value of α^* is found by representing $\phi(\alpha)$, in the bounded interval, by a cubic polynomial. This cubic polynomial is refitted in the fourth stage, if the value of α^* found in the third stage does not satisfy the convergence criteria. A better point in the design space is then found as

$$\overline{X}_{i+1} = \overline{X}_i + \alpha_i^* \, \overline{S}_i \, . \qquad ..(6.2)$$

(9)   The new point $\overline{X}_{i+1}$ is tested for optimality. If $\overline{X}_{i+1}$ is optimal, the iterative procedure is terminated.

(10)   Otherwise, the $[H]_i$ matrix is updated as

$$[H]_{i+1} = [H]_i + [M]_i + [N]_i , \qquad ..(6.3)$$

where $\quad M_i = \dfrac{\bar{S}_i \bar{S}_i^T}{\bar{S}_i^T \bar{Q}_i} ,$

$$N_i = -\dfrac{([H]_i \bar{Q}_i)([H]_i \bar{Q}_i)^T}{\bar{Q}_i^T [H]_i \bar{Q}_i} , \qquad ..(6.5)$$

and $\quad \bar{Q}_i = \nabla \phi_{i+1} - \nabla \phi_i .$ $\qquad ..(6.6)$

(11) The new iteration number i is set equal to i+1 and the new iteration commenced from step (7).

6.3 Newton-Raphson Method

There are several numerical methods available to find the roots of polynomial and transcendental equations of a single variable. Newton-Raphson method can be considered to be based on truncation of the Taylor series. If x_i is an approximate root of the equation $f(x) = 0$, then

$$f(x_i) + hf'(x_i) + \frac{h^2}{2!} f''(x_i) + \ldots = 0 , \qquad ..(6.7)$$

where the prime denotes the order of differentiation and h the difference between the actual root and x_i. Neglecting higher order terms, a better approximation to the root, x_{i+1}, can be obtained from the relationship

$$f(x_{i+1}) = f(x_i + h) \simeq f(x_i) + hf'(x_i) \simeq 0 , \quad ..(6.8)$$

from which

$$h = -\frac{f(x_i)}{f'(x_i)}, \quad f'(x_i) \neq 0 \text{ for all } i. \qquad ..(6.9)$$

Thus the following iteration scheme is obtained,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}. \qquad ..(6.10)$$

The success of this method depends on the choice of the initial point. If a proper starting point is chosen, the rate of convergence is quite fast.

The Newton-Raphson method can be applied with equal facility for the solution of a system of nonlinear equations, although the computational effort will be substantially more. In this case, the iteration scheme will be

$$\overline{X}_{i+1} = \overline{X}_i - [J]_i^{-1} \overline{E}_i, \qquad ..(6.11)$$

where $\overline{X}_i$ = vector of the variables at the end of i

iterations,

$[J]_i$ = Jacobian matrix evaluated using $\overline{X}_i$,

and $\overline{E}_i$ = error vector (vector of the right hand side of the system of equations, evaluated using $\overline{X}_i$).

While a system of linear simultaneous equations has a unique solution, if the coefficient matrix is non-singular, a set of nonlinear simultaneous equations may have multiple solutions. Therefore, great care has to be exercised in choosing the initial vector $\overline{X}_o$, as otherwise

the solution may either converge to a vector outside the domain of interest or not converge at all.

6.4 Organization of the Computer Programs

The computer programs used in this study can be broadly classified into three categories; main program, optimization subroutines and function subroutines. The main program essentially reads the input data, calls the relevant subroutines, and prints the output. The principal subroutines employed to carry out the optimization are SEARCH and STEP, the former to find out the search direction and the latter to obtain the minimizing step length. In order to find the gradient of the modified objective function and the directional derivative, these subroutines in turn call GRAD, which makes use of a finite difference approach to find the gradient vector. Since the gradient, in the finite difference scheme, is computed on the basis of function values, subroutine GRAD in turn calls FUNCTN. Thus, SEARCH, STEP, and GRAD constitute the optimization subroutines and these are used for all the problems in the same form. Only subroutine FUNCTN changes from problem to problem.

The purpose of FUNCTN is to find the values of the objective function F, all the constraints g_j (j = 1, 2,... n), and the modified objective function $\emptyset$ for a chosen value of the penalty parameter. For the T-beam floor problem, all the constraints and the objective

functions are evaluated in the FUNCTN subroutine only. In the case of water tank problems, additional subroutines SERVIS and LIMIT are called by FUNCTN in order to carry out the elastic analysis and limit analysis of the tank. These in turn make use of the following subroutines: GAUSS to solve a set of linear simultaneous equations, NEWTON to solve polynomial or transcendental equations of a single variable, RAPSON to solve a set of nonlinear simultaneous equations, JACOB to assemble the Jacobian matrix, and MATINV to obtain the inverse of a matrix. Function subprograms are employed by some of these subroutines to find the value of functions or their derivatives.

6.5   Implementation of the Optimization Programs

Several precautions are necessary for a satisfactory implementation of the optimization programs. The following are among the important ones:

(1)   Normalization of the design variables, the constraints and search directions, which have been mentioned earlier.

(2)   The contours of the modified objective function will be distorted and tend to be eccentric, even if the objective function is well behaved, especially when the algorithm is initiated and the penalty parameter has a high value. It is possible to change the shape of contours, by a suitable scaling of the

design variables, such that the algorithm gets initiated and converges to the optimum solution rapidly.

(3) The penalty parameter r and reduction factor c have to be properly chosen. An initial penalty of 50% to 100% of the value of objective function at the starting point and a value of 0.1 for c have been found to be satisfactory.

(4) In almost all constrained optimization problems, there will be some constraints which are critically satisfied. Since the penalty term tends to infinity as the solution approaches the constraint boundaries, special precautions have to be taken to overcome the overflow problem.

(5) Several convergence criteria have to be incorporated in order to avoid a premature termination. These could include comparison of the relative values of the objective functions, the design variables, the gradients and related quantities, of two successive iterations. In order to make use of the solution so obtained with confidence, and to test for optimality, perturbation of the design vector and testing the Kuhn-Tucker conditions can be resorted to.

(6) All the techniques available for the solution of nonlinear problems guarantee, at best, a local minimum. This would also be the global minimum, if the objective

function is convex. It is not easy to ascertain whether the objective function is convex or not, for the nature of functions encountered in this study or in other real life problems. To get over such a situation, the optimum solution is obtained with different starting points and the solutions compared; if the solutions match then it is very likely, but not certain, that the solutions correspond to the global minimum.

The Table 6.1 shows the optimal designs of a cylindrical surface water tank with a domed roof, obtained from three different starting points. It can be seen that there is no significant difference in the solutions obtained. In the current study, since a number of problems of the same nature, but with different data, have been solved, the possibility of having missed the global minimum is practically nil.

A sort of reverse situation, that of multiple optima, has been noticed in the solution of the T-beam floor problem, because of the comparatively flat surface of the objective function in the h_b (depth of beam) plane. This is illustrated in Table 6.2. As a consequence, in comparison with the optimal values of h_b or A_{st} for a particular level of moment, smaller value of one of the variables with a consequent increase in the other, has been sometimes indicated for a higher value of moment. In order to obtain

Table 6.1   Optimal designs of a water tank from different starting points

Capacity = 500 m^3, β = 1.0, R_1 = 1200, R_2 = 100

Nomenclature	R (mm)	t (mm)	a_{v2} (mm^2/m)	a_{h2} (mm^2/m)	Objective function
First starting point	8000.0	150.0	1500.0	3000.0	-
Optimal design 1	5682.3	126.4	773.2	1638.6	153771
Second starting point	5000.0	250.0	2000.0	2500.0	-
Optimal design 2	5664.6	126.1	792.4	1653.3	153784
Third starting point	6000.0	200.0	1000.0	4000.0	-
Optimal design 3	5671.3	125.7	778.5	1641.2	153762

Table 6.2   Multiple optima for a T-beam floor problem

$l_s = 4000$ mm, $l_b = 8000$ mm, $q_k = 3$ kN/m^2, $R_1 = 1248$, $R_2 = 50$

solution No.	h_s (mm)	h_b (mm)	a_{s1} (mm^2/m)	a_{s2} (mm^2/m)	a_{s3} (mm^2/m)	A_{st} (mm^2)	Objective function
1	119.8	547.2	637	358	216	1713	1122.9
2	122.4	511.0	556	363	214	1857	1129.2
3	121.0	573.0	600	365	211	1625	1125.0
4	118.4	523.8	709	356	217	1779	1122.0

a smooth curve to relate these variables with the moment, this situation necessitated a few more function evaluations.

6.6 Stability of the Optimal Solutions

If the optimal designs are very sensitive to small departures in the value of design variables from the optimum, then the usefulness of the results will be limited. This is because, when the results of the optimization studies are applied to the actual situation, values of design variables may have to be chosen slightly different from the optimum, due to considerations other than economy. Therefore, it is important to investigate the effect of such departures from the optimum design on the objective function.

This aspect has been considered in the current study, though not in detail. It is expected that the design charts developed will be essentially useful to pick optimal values of thickness of slab, depth of beam, or radius of tank wall. Then, based on the guide lines given for choosing the rib width or thickness of tank wall, areas of various reinforcements can be obtained using the limit state theory. Therefore, only effect of variation of slab thickness, depth of beam, and radius of tank wall, on the objective function, has been studied. The procedure that has been followed is to vary these quantities from their optimum values, one at a time, by a predetermined magnitude and then again optimize the design with the rest of the

design variables. As such, these are also optimal designs, but with a preassigned value for one of the design variables. These optimal designs have been obtained for the middle cost ratio combination using concrete of grade 25.

If h_s^* and F^* correspond to the optimal values of the thickness of slab and objective function for the T-beam floor, effect of choosing a slab thickness different from h_s^*, on F^*, has been shown in Figure 6.1. The chosen thicknesses for the slab are $0.975 \ h_s^*$, $1.05 \ h_s^*$, $1.1 \ h_s^*$, $1.15 \ h_s^*$ and $1.2 \ h_s^*$. The thickness of the slab cannot be reduced to even $0.95 \ h_s^*$ inspite of increasing a_{s1}, area of reinforcement in the middle portion of end span. This is because, the modification factor ξ associated with the allowable span to depth ratio for the slab, depends both on the stress in the reinforcement at service loads and the percentage of reinforcement; increase of a_{s1} on the one hand leads to a decrease in the value of the former, and on the other an increase in that of the latter. Therefore, beyond a certain limit, the value of ξ cannot be increased even with an increase in the value of a_{s1}.

With the chosen thicknesses of the slab, results have been obtained for 5 problems and the average represented in Figure 6.1. It is observed that the objective function value increases by less than 2% when the slab thickness is $1.05 \ h_s^*$ and by about 4.2% when the slab thickness is $1.1 \ h_s^*$. Therefore, it is better to choose the thickness of slab to

be close to the optimum value. On the other hand, the
objective function is very insensitive to departures in the
depth of beam from the optimal as shown in Figure 6.2. The
solid line corresponds to the average value of results of
6 problems, 3 chosen from lower part of M against h_b curve
(Figure 2.7) and 3 from the upper part. The dotted line
corresponds the average value of the results of 4 problems
chosen in the middle part of this curve, which have all an
optimal beam depth of 750 mm. In this case, when the depth
of beam is decreased, the increased area of reinforcement
demands a substantial increase in the value of cover to
reinforcement and hence the increase in the value of objective
function is comparatively more. For these 4 problems,
increase in the value of h_b is not considered as it will
result in provision of side face reinforcement.

Effect of choosing non-optimal radii on the objective
function of surface cylindrical tanks, with or without a roof,
is shown in Figure 6.3. For each of these categories, 3
different capacities, each with 3 different degrees of
fixity, have been considered. From the curves obtained with
the averaged results, it is seen that for a 10% departure
in the value of radius from the optimum, the increase in the
value of objective function will be less than 2%; this value
increases rapidly beyond a departure of 15% from the optimum
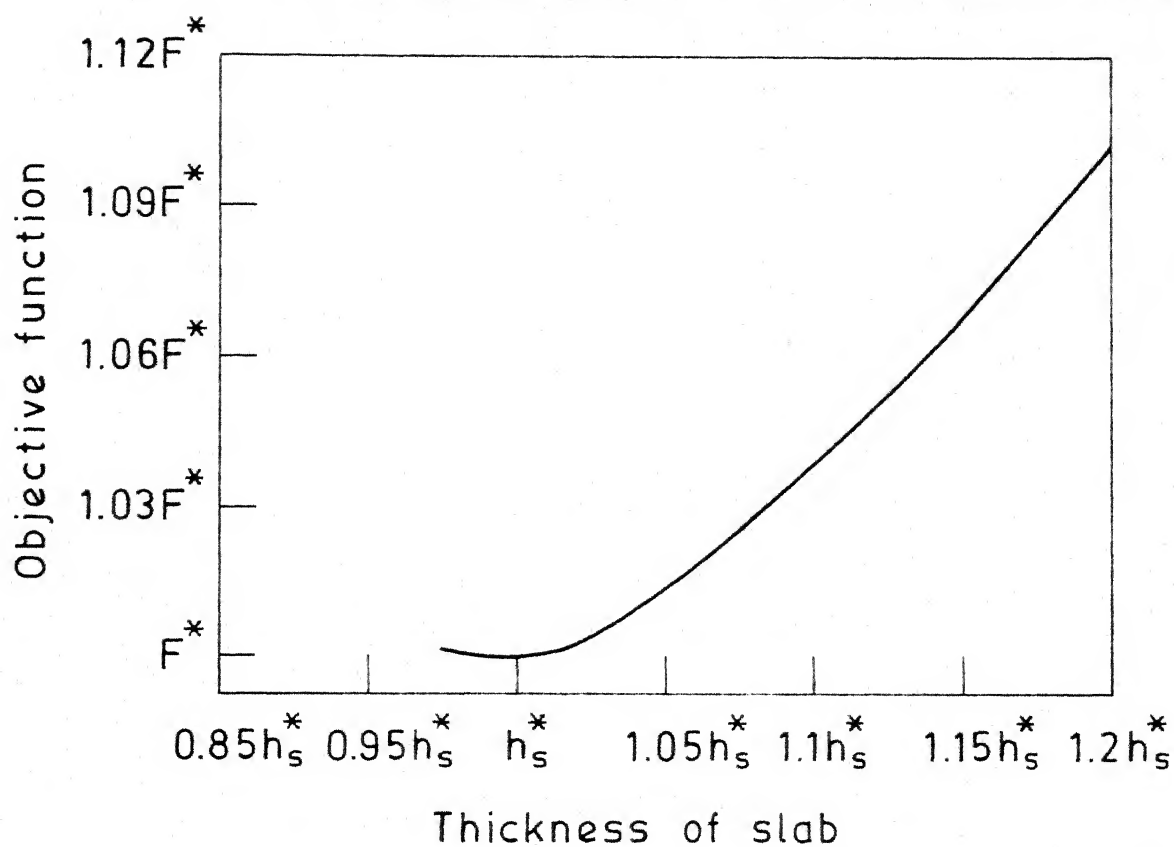radius.

Fig. 6.1 – Effect of non-optimal slab thickness on the objective function.

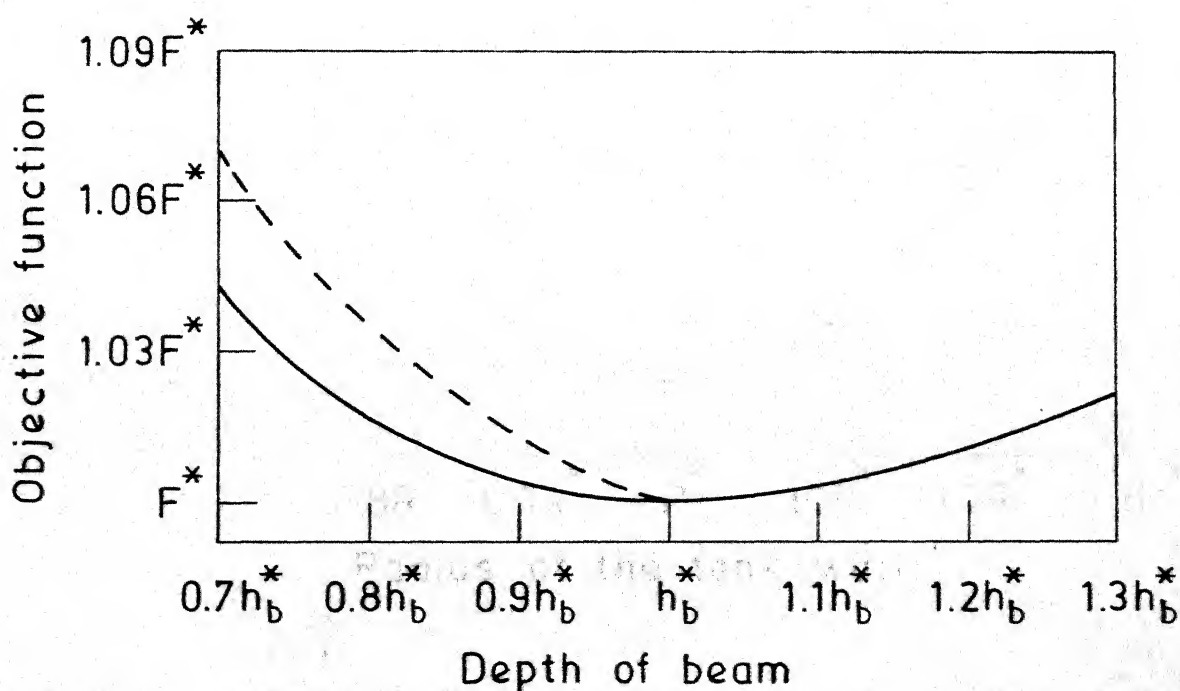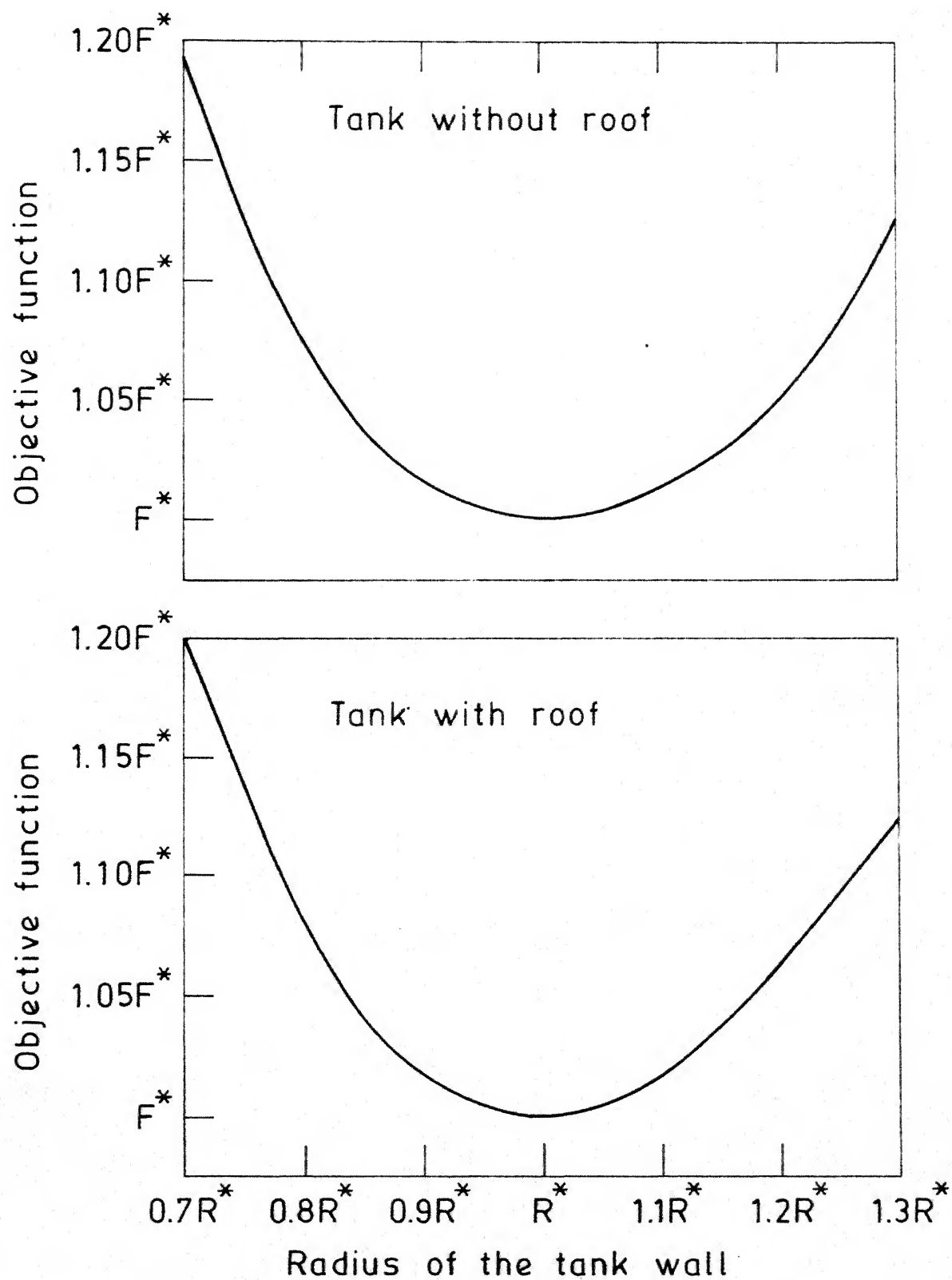Fig. 6.2 – Effect of non-optimal beam depth on the objective function.

Fig. 6.3 - Effect of non-optimal radius on the objective function.

Thus, it is seen that the thickness of slab is a somewhat sensitive variable while the depth of beam or area of tension reinforcement for the beam is not. Objective function of the tank is sensitive only to large departures from the optimum value of the radius of tank wall.

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1   General

Optimal limit state designs of some reinforced concrete structures, conforming to CP 110 and BS 5337, have been considered in this study. As pointed out in the beginning, the methodology developed can be gainfully applied for the optimal designs of slabs, beams, and storage structures like water tanks and silos; T-beam floors and cylindrical water tanks have been studied in greater detail and design charts developed for their optimal design.

Optimization has enabled in choosing those values of design variables which would minimize the cost of the structure and limit state philosophy in ensuring, during the process of systematic search, satisfactory functioning of the structure at service as well as ultimate loads. The wide range of cost ratios considered has made identifying variables which are sensitive or insensitive to variations in the cost ratios possible.

The following are some of the broad conclusions that are applicable for optimal design of reinforced concrete structures in general:

(1)   Strengths of concrete mixes, of those that are normally employed for the construction of reinforced concrete structures, do not significantly affect

either the dimensions or cost of the structures. As such, it is better not to choose mixes which are particularly rich in cement content in order to avoid the associated thermal cracking problem.

(2)   In view of their superior bond characteristics, high yield strength deformed bars are always preferable to mild steel reinforcement, even in circumstances which do not permit a high stress in them. If advantage of their higher strength can be taken into account, as is normally the case, resulting economy will be substantial.

(3)   The thicknesses of slab or wall like structural members, tend to be as small as the functional or other considerations permit; in order that behaviour constraints pertaining to strength or serviceability of the structure are not violated, additional reinforcement is indicated around critical sections.

(4)   The optimal values of thickness, and even reinforcements in the case of slabs, will be practically independent of cost ratios of materials for such structural members.

(5)   The saving in the use of important construction materials, cement and steel, through use of optimal designs, contributes towards meeting the present crisis of shortage of building materials (Patel, 1981).

7.2   T-beam Floors

The following conclusions are in particular applicable to the optimal design of T-beam floors:

(1)   Limit state of deflection governs the optimal design of the slabs. It would be an optimal policy to provide more reinforcement in the end span of a continuous slab, than is required from the strength point of view, in order to arrive at thinner and more economical slabs.

(2)   The area of main reinforcement to be provided over the supports, a_{s2}, has been invariably found to be equal to the minimum value of 0.3% specified for effective T-beam action. From strength consideration, such a value is not required in many cases. Therefore, this code provision needs to be examined, especially in view of the high strength of reinforcing bars.

(3)   The wide flange of T-beam provides the required compressive strength and it has been found that use of compression reinforcement is neither required nor optimal.

(4)   The rib for the beam need not be wide even from shear force considerations. In fact, it would be economical to provide as narrow a rib as practical considerations permit. Thus, grouping of bars in the vertical direction (preferably 2 bars in one group) and providing more than one layer of such

reinforcement, when required, is found to be more advantageous than providing a wider rib to accommodate more number of bars per layer.

(5) While making use of the design charts, it is better to choose a thickness of slab as close to the optimal as possible. Since the value of a_{s1}, reinforcement in the central portion of the end span, is sensitive to changes in the thickness of slab, it would be advisable to find the required value of a_{s1} from the deflection consideration whenever the chosen thickness is different from the optimum. The objective function of the T-beam floor is seen to be insensitive to departures of the depth and area of tension reinforcement, for the beam, from the optimal values.

(6) Although the deflection and crack width criteria do not normally affect the optimal design of T-beams in view of their comparatively large depths, control of crack widths comes into prominenece for beams which have an overall depth greater than 750 mm. In order to control the width of cracks on the sides of such beams, the code specifies that side face reinforcement has to be provided for two-thirds of their depth. Provision of such a reinforcement causes a sudden jump in the value of objective function, at that point where the beam depth crosses the 750 mm mark, which acts as a sort of barrier. So, over a considerable

range of values for the moment at ultimate loads, the depth of beam stays constant at 750 mm, the increased moment being resisted by an increase in the value of tension reinforcement only. Depths beyond 750 mm are optimal only for beams which have large spans with heavy loads. As such, 750 mm may be considered to be an optimal upper limit on the depth of beam for the normal class of T-beams.

(7)   For analysis purposes a 3-span one-way continuous slab has been assumed for this study. This does not preclude application of the results obtained to situations in which the number of spans are different. In fact, the moment coefficients do not vary much if the number of spans is more. Even otherwise, the thickness of slab and the area of reinforcement in the end span are governed by the moment at service loads in the centre of end span and as such are practically independent of the number of spans. The area of reinforcement over the supports being governed by the minimum value remains unaltered. Thus, only the areas of reinforcement in the central portion of intermediate spans depend on the number of spans and can be easily computed for the different cases once the thickness of slab has been arrived at.

7.3   Water Tanks

The following conclusions pertain to the optimal design of water tanks:

(1)   The design of water tanks is considerably influenced by the code provisions to achieve watertightness which is its most important serviceability requirement. The working stress approach to the design of water tanks, requiring the thickness of concrete members to be based on the permissible tensile stress in concrete, is seen to be both irrational and conservative. Application of limit state theory results in smaller thicknesses for the concrete members as well as lesser areas of reinforcement.

(2)   Design of cylindrical surface water tanks is conventionally based either on a hinged or a fixed base condition, although it is recognized that the actual restraint at the base is likely to be in between the two conditions. By the method of analysis developed in this study, any degree of fixity can be assigned. Thus, if a proper estimation of the degree of fixity can be made, based on the properties of soil and other factors that influence it, a more realistic distribution of moments and hoop tensions in the tank wall can be obtained.

(3)   It is found that the forces in the domed roof do not warrant a 100 mm thickness, which is commonly the

minimum thickness provided based on the constraints imposed by the construction process. A ferrocement dome with a fine chicken mesh can be easily constructed to have a smaller thickness and is better from the crack control point of view also. If found suitable, it can lead to a significant reduction in the cost of the structure.

(4) It is seen that the stress in reinforcement due to flexure can be as high as 275 MPa without violating the flexural crack width constraint; on the other hand, the stress in hoop reinforcement is restricted to 130 MPa for class B exposure based on the 'deemed to satisfy' provision. Thus, the need for further research, to explore the possibility of utilizing the strength of reinforcement in members subjected to direct tension without violating the crack width constraint, is strongly felt.

(5) The optimal radius of the tank wall is essentially influenced by the cost ratio R_1, except in the case of surface water tanks without roof for which the influence of cost ratio R_2 is also significant. Effect of degree of fixity is seen to be marginal.

(6) The optimal radius is not a very sensitive variable and departures of the order of 10% from the optimum do not increase the value of objective function considerably.

(7) Overhead cylindrical tanks, with a flat bottom and an annular support of a smaller diameter, can be designed with a substantially thinner floor slab and lesser reinforcement by the limit state theory. Therefore, this type of tank may be used with advantage for both small and medium capacities rather than those with complicated bottoms.

(8) The mode of failure of the tank wall, in the limit analysis, is by the partial collapse of its bottom portion for the different types of water tanks considered in this study. The factor of safety against collapse is observed to be higher than the required value of 1.6.

(9) The strength constraint becomes active only in the design of the floor slab of the overhead tank. Since the magnitude of water pressure, which is treated as imposed load on the floor slab, is known precisely, the specified partial safety factor of 1.6 for it is irrational and needs reconsideration.

7.4   Suggestions for Future Work

The following are some of the suggestions which may be considered for future work:

(1) Optimal design of reinforced concrete structures in a reliability framework will enable specified risk levels to be incorporated in the study.

(2) The study of T-beam floors can be extended to include two-way slab as well as continuous beam systems.

(3) It can also be extended for the design of T-beam bridges using either reinforced or prestressed concrete beams.

(4) Extension of the present work for the study of underground tanks and silos is straightforward.

(5) Study of optimal limit state design of the different types of tanks using a rectangular shape will enable comparisons to be made between the designs employing the two shapes.

(6) Optimal designs of overhead tanks, of different shapes and for different capacities, carried out over a wide range of cost ratios, will help in choosing the most economical shape under the prevailing circumstances.

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